Constraining the Two Higgs Doublet Model with CP-Violation

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Contents

Chapter 1

Introduction

Recently, high energy physicists have arrived at a picture of the microscopic physical universe, called "The Standard Model" (SM) which unifies the nuclear, electromagnetic, and weak forces and enumerates the fundamental building blocks of the universe. The mechanism of electroweak symmetry breaking is the most important ingredient in the description of elementary particles physics. The SM incorporates the Higgs mechanism that breaks the electroweak symmetry spontaneously through a neutral scalar field with a non-zero vacuum expectation value (v.e.v.). In the minimal version of this mechanism one scalar $SU(2)_L$ doublet is required, providing one physical particle (the Higgs particle) [1–4]. The Higgs particle is a hypothetical massive scalar elementary particle predicted to exist by the SM of particle physics. The Higgs boson field is the mechanism which extends the SM to explain how particles acquire the property of mass. The Higgs boson is the exchange particle in this field and it is not observed yet.

CP violation is one of the crucial ingredients necessary to generate the observed matter-antimatter asymmetry of the Universe: It is not possible to generate a baryon asymmetry of the observed size with the very small CP violation present in the SM [5] via the complex phase in the CKM matrix. New sources of CP violation in models beyond the SM can play an important role in the explanation of the observed size of this asymmetry.

One of the most popular extensions of the SM is the Two-Higgs doublet Model (2HDM), which is formed by adding an extra complex scalar doublet to the SM. As a consequence there exist a variety of new sources of CP violation, which are required to explain the matter-antimatter asymmetry in the Universe (baryon asymmetry) [6]. Various motivations for adding a second Higgs doublet to the SM have been advocated in the literature [7–10]. The quantity $\rho = M_W/(M_Z \cos^2\theta_W) = 1$ like the SM at tree level [11,12], if both Higgs fields are weak isodoublets ($T = 1/2$) with hypercharge $Y = 1$. After the electroweak-breaking mechanism, three of the eight degrees of freedom of the two complex scalar doublets are absorbed by the $W^\pm$ and Z^0 bosons to be their longitudinal components. The remaining five degrees of freedom form five elementary Higgs particles. The physical spectrum of the 2HDM contains five Higgs bosons: two neutral scalars (CP-even) h (the lightest one corresponding to the SM Higgs) and H (the heaviest one), and the neutral pseudoscalar A (CP-odd) and two charged scalars $H^\pm$ in the case of CP conservation [13]. In the most general CP-violating 2HDM, the physical Higgs fields are linear combinations of h, H and A. A non-zero neutron electric dipole moment

(NEDM) is a consequence of CP violation and can arise through the exchange of the neutral Higgs bosons [14]. Also Weinberg proposed a gauge theory of CP non-conservation through the exchange of the charged Higgs bosons [15]. These particles could be detected directly at the LHC or indirectly through their contributions as intermediate states in decay process.

The 2HDM presents a richer phenomenology due to the appearance of the charged and two more neutral Higgs particles. Many studies advocated to constrain the parameters of the 2HDM.

It is convenient to split the constraints into three categories:

(i) Theoretical consistency constraints: positivity of the potential [16–18] and perturbative unitarity [17, 18, 20–22]. From the theoretical point of view, there are various consistency conditions. The potential has to be positive for large values of the fields. We also require the tree-level Higgs-Higgs scattering amplitudes to be unitary. Together, these constraints dramatically reduce the allowed parameter space of the model.

(ii) Experimental constraints on the charged-Higgs sector. These all come from B-physics, and are due to $b \to s\gamma$, B–$\bar{B}$ oscillations, and $B \to \tau\nu_\tau$ [17, 19]. They are all independent of the neutral sector.

(iii) Experimental constraints on the neutral sector. These are predominantly due to the precise measurements of the partial decay width R_b for the process $Z \to b\bar{b}$, non-observation of a neutral Higgs boson at the Large Electron-Positron collider (LEP2), the precision measurements of the parameter $\Delta\rho$ which measures the deviation of the W and Z self-energies from the standard model value, and $a_\mu = \frac{1}{2}(g-2)_\mu$ [17,19].

The first and third categories of constraints will depend on the neutral sector, i.e., the neutral Higgs masses and the mixing matrix. The second category is due to physical effects of the charged-Higgs Yukawa coupling in the B-physics sector. These are "general" in the sense that they do not depend on the spectrum of neutral Higgs bosons, i.e., they do not depend on the mixing (and possible CP violation) in the neutral sector.

When considering the different experimental constraints, our basic approach will be that they are all in agreement with the Standard Model, and simply let the experimental or theoretical uncertainty restrict possible 2HDM contributions (this procedure yields lower bounds on the charged-Higgs mass, possibly also other constraints). An alternative approach would be to actually fit the 2HDM to the data.

The thesis is organized as follows:

In Chapter 2, we present some basics of the SM theory such as symmetries, gauge field theory. The spontaneous symmetry breaking is discussed and the Higgs mechanism is presented. Also the limitations on the SM Higgs mass are mentioned.

The 2HDM theory is discussed in Chapter 3. The 2HDM Lagrangian and Yukawa coupling are discussed. The CP conservation and CP-violation in the 2HDM are presented. The general potential of the 2HDM is studied. The Yukawa coupling of the neutral Higgs boson with top and bottom quarks are presented. Finally the SM-like Higgs boson for the CP conserving case is studied.

The theoretical constraints on the 2HDM are studied in Chapter 4. The positivity for the CP non-conserving case is presented. The unitarity constraint for both cases of CP conservation and CP non-conservation are studied.

In Chapter 5, the experimental constraints on the charged Higgs sector are presented. The constraints from the B-physics such as the $\bar{B} \to X_S\gamma$ constraint, the constraint of the $\mathcal{B}(\mathcal{B}^- \to \tau^-\bar{\nu}_\tau)$ decay and $B-\bar{B}$ oscillations are studied. The constraints from the neutral Higgs sector are also presented, such as, the anomalous magnetic moment of the muon, the R_b constraint, the constraint coming from electroweak correction to the ρ parameter and the LEP2 non-discovery.

The summary and conclusion are presented in Chapter 8.

Four publications are presented in an Appendix:

- Paper 1 "Consistency of the two Higgs doublet model and CP violation in top production at the LHC", Nucl. Phys. B775:45-77, 2007.

- Paper 2 "Constraining the Two-Higgs-Doublet-Model parameter space", Phys. Rev. D76:095001, 2007.

- Paper 3 "CP violation, Stability and Unitarity of the two Higgs doublet model", Nonlin. Phenom. Complex Syst. 10:347-357, 2007.

- Paper 4 "Profile of two-Higgs-doublet-model parameter space", Talk given at 2007 International Linear Collider Workshop (LCWS07 and ILC 07), Hamburg, Germany, 30 May-3 Jun 2007.

Chapter 2

The Standard Model

The Standard Model (SM) of particle physics has been one of the most successful achievements of modern physics. Within a simple and elegant framework, it perfectly describes most of the collected data so far. The standard model explains the fundamental particles and their interactions. The electroweak theory $SU(2)_L \times U(1)_Y$ was first proposed by Glashow, Salam and Weinberg [23]. The theory of strong interactions between colored quarks described by Quantum Chromo-Dynamics (QCD) [24] is based on the symmetry group $SU(3)_C$. The SM theory combines the electroweak and strong interactions based on the local $SU(2)_L \times U(1)_Y \times SU(3)$ gauge group. The electroweak gauge fields $W^{\pm}$, Z and the photon field A^{μ} correspond to the four generators of the non-Abelian $SU(2)_L \times U(1)_Y$ and the color group $SU(3)$ has eight generators associated with the equivalent number of massless gluons. The Standard Model being a gauge theory implies that its Lagrangian is invariant under certain types of symmetry transformations, so that the theory should be regularized [25] (to evade the divergence) and renormalized [26].

The SM is built upon two types of particles, fermions (half-integer spin) which are divided into leptons and quarks (the building blocks of matter and anti-matter of the universe), and bosons (integer spin) which mediate the interactions between particles. The interaction between matter and gauge fields is incorporated into the theory by minimal substitution which amounts to replacing the partial derivatives in the Lagrangian with the covariant ones including the couplings related to the various gauge groups.

There are four different forces in nature, corresponding to the exchange of four types of fields, the weak force is mediated by the massive weak bosons $W^{\pm}$, Z^0 which are responsible for the nuclear decay, the electromagnetic force is mediated by the massless photon, the strong force is mediated by eight massless gluons carrying colors which bind the quarks, and the gravitational force is mediated by the graviton and responsible for the gravity.

High-precision measurements at LEP, SLC, Tevatron [27,28] have provided a decisive test of the standard model and firmly established and provided a clear description of the strong and electroweak interaction at the present energies [29]. These tests, performed at the per mille level accuracy, have probed the radiative corrections and the structure of the $SU(2)_L \times U(1)_Y \times SU(3)$ symmetry, and the precise measurements of the couplings of the gauge bosons with quarks and leptons agree with the theory i.e., the discovery of the $W^{\pm}$, Z at CERN [30], except that the Higgs bosons until now has not observed [27].

2.1 A gauge theory of weak interaction

The exact conservation laws reflect the fact that nature has exact symmetries, i.e., the conservation of energy, momentum, angular momentum (because of the invariance of forces under rotation in time and space, respectively) and the electric charge. These conservation laws or exact symmetries require two conditions. The Lagrangian density is invariant under the symmetry $\delta\mathcal{L} = 0$ and there is a unique vacuum state (the ground state is not degenerate) [31].

To understand the electroweak sector of the standard model, one tries to understand the different kinds of symmetries and the elements of the gauge theory.

2.1.1 Symmetries

Symmetry is close to harmony, beauty and unity, and means that under certain transformations of a physical system, aspects of this system are shown to be unchanged. The symmetry properties of a physical system are related to the conservation laws characterizing this system, according to Noether's theorem, each symmetry of a physical system implies that some physical properties of that system are conserved [32]. These symmetries may be continuous or discrete.

Discrete Symmetries

A discrete symmetry describes non-continuous changes in a system, and it also simply flips a system from one state to another, such as spatial symmetry or parity inversion (P), which corresponds to a spatial reflection of physics through the coordinate origin, charge conjugation (C) which connects particle and antiparticle, and time reversal (T) which reverses a given physical process in time. Each individual symmetry can be violated naturally but physical laws must be invariant under CPT (the combinations of the three transformations C, P, and T which are called general symmetry).

This transformation performs a reflection of the space axes through the origin, inverts the time evolution, and interchange particles and antiparticles. CPT symmetry can be used to show that the particle and anti-particle must have certain identical properties including mass, lifetime, and the size of charge and magnetic moment [33]. Optimistic scientists still believe that CPT is not broken (the mirror image of the antimatter world with time running backward should look exactly the same as ours) but if CP is violated it must be compensated by time reversal violation. T and CPT transformations are anti-unitary and interchange outgoing with ingoing states.

Another important symmetry is CP (combinations of C and P). CP symmetry implies that particles and antiparticles behave like mirror images of each other. There is strong evidence that the universe is composed mostly of matter rather than of antimatter. It is believed that the particles and anti-particles were equally numerous in the early universe, but the particles became dominant as the universe cooled. This phenomenon is called the baryon asymmetry, and the theory to describe it is called baryogenesis [34]. Antimatter in our universe is necessarily very short-lived because of the overwhelming preponderance of ordinary matter. To explain the observed baryon asymmetry, Andrei Sakharov postulated that three requirements [35] must be fulfilled: that the universe must be out of equilibrium, the conservation of baryon symmetry must be violated and CP symmetry also must be violated (otherwise any process that changes the amount of matter would be balanced by a similar effect for antimatter) [36].

There are three ways to introduce CP violation.

- One of them occurs when quarks undergo weak interactions and turn into quarks with different electric charge and this type can be represented by the complex phase of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [37]. A pure gauge Lagrangian is CP-invariant, the scalar potential of the SM conserves CP, but CP violation can arise from the presence of Yukawa interactions and gauge interactions. The CKM matrix V is complex, but some of the phases in it do not have physical meaning, so one has the freedom to rephase some quark fields. In SM with n_g generations the CKM matrix is $n_g \times n_g$ unitary, it is parameterized by n_g^2 parameters, but $2n_g - 1$ phases are absorbed by rephasing the quark fields. Therefore the number of physical parameters of V are [38]

  $$N_{\text{param}} = n_g^2 - (2n_g - 1) = (n_g - 1)^2 \tag{2.1}$$

  The number of rotation angles (Euler angles) used to parametrize an $n_g \times n_g$ orthogonal matrix is given by

  $$N_{\text{angle}} = \frac{1}{2} n_g (n_g - 1) \tag{2.2}$$

  Therefore the number remaining physical phases are

  $$N_{\text{phase}} = N_{\text{param}} - N_{\text{angle}} = \frac{1}{2}(n_g - 1)(n_g - 2) \tag{2.3}$$

  The CKM matrix can be written after parametrization as

  $$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \tag{2.4}$$

  where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$ for $i = 1, 2, 3$.

  For three generations in the SM, CP violation comes only from one complex phase. The CP violation in the SM is proportional to the Jarlskog parameter [39]

  $$J = \text{Im}\,(V_{us} V_{cb} V_{ub}^{\star} V_{cs}^{\star}) \tag{2.5}$$

  In general any Lagrangian under CP transformations can be written as

  $$\mathcal{L} = \mathcal{L}_{CP} + \mathcal{L}_{\text{remaining}} \tag{2.6}$$

  where $\mathcal{L}_{CP}$ is CP conserving under CP transitions but $\mathcal{L}_{\text{remaining}}$ is not.

- CP violation is also possible in the mixing of leptons.

- CP violation could also be present in the strong interactions, as a result of non-perturbative effects, but this has not been seen. Strong CP violation can occur if one adds a new term to the SM Lagrangian $\mathcal{L}_\theta = -(\theta/32\pi^2)F_{\mu\nu}\tilde{F}^{\mu\nu}$ which can contribute to the neutron electric dipole moment d_n ($\theta < 3 \times 10^{-10}$ is very small). The problem of why θ is so small is known as the strong CP problem [40]. CP violation occurs as a result of the electroweak symmetry breaking, therefore it can be used to probe the energy scale of new physics [41].

The first time CP violation was observed in laboratories was by Val L. Fitch and James W. Cronin in 1964, they studied the decay of neutral kaon particles. They found that the indirect CP violation or CP violation in the mixing (CP violation in $\Delta S = 2$ transitions) exists in kaon decay and they shared the Nobel prize in physics in 1980 for this discovery [42]. The existence of direct CP violation or CP violation in the decay amplitudes (CP violation in $\Delta S = 1$ transitions) in the neutral kaon was first observed at CERN (NA31) [43], and confirmed at Fermilab (KTeV experiment) and at CERN (NA48 experiment) [44,45]. Another one is called interference CP violation which occurs between the mixing and decay amplitudes.

The SM can not explain the baryon asymmetry that exists in the universe because there is a very small CP violation that can be produced in the SM. Therefore new physics must be introduced to provide more sources and larger amount of CP violation to explain the baryogenesis. One promising model is the Two Higgs Doublet Model (2HDM), which can introduce CP violation via complex coupling constants of the quadratic and quartic terms of the Higgs potential, as we see in the next chapter.

Continuous Symmetries

A continuous symmetry is characterized by a continuous change in the geometry of the system, and mathematically represented by a continuous function, which can be divided into two types of symmetry. Internal symmetry as well as space-time symmetry:

- Space-time symmetries. These symmetries include the Lorentz and Poincare groups and are related to space-time, such as time translation, spatial translation, spatial rotation, etc. Fields are classified under these symmetries as scalar fields, vector fields, tensor fields, and spinors.

- Internal symmetries. The simplest internal symmetry corresponds to the possibility of re-phasing each individual quantum field. More generally, whenever there are various quantum fields with the same quantum numbers, there is an internal symmetry mixing those fields. That symmetry is unitary, because the kinetic terms of the Lagrangian should preserve their renormalization [38]. Internal symmetries mix particles among each other, i.e., transform one particle into another with different internal quantum numbers but with the same mass [48,49]. Heisenberg in 1932 [50] was the first one to suggest that under nuclear interactions, the proton and neutron can be regarded as degenerate, since their masses are quite similar and the electromagnetic interaction is negligible, i.e., the proton and neutron are considered as an SU(2) isospin doublet.

In particle physics there are many examples of these symmetries such as the color symmetry of the interactions of quarks, $SU(N)$ isospin or flavor symmetry.
The internal symmetries can be either

- Global or phase symmetries, which are independent of space-time, i.e., hold at all points of space and are defined as:

$$\psi(x) \rightarrow \exp(-iqf)\psi(x) \tag{2.7}$$

where q is the charge and f is a constant.

- Local symmetries, which are dependent on space points and are more general, i.e., the symmetry group varies at each space-time point. The idea of local gauge isotopic invariance in quantum field theory was introduced by Yang and Mills [51], according to them, the differentiation between a proton and a neutron is a purely arbitrary process but this arbitrariness is subject to the following limitation: once one chooses a proton and a neutron at one space-time point, one is then not free to make another choice at another space-time point.

Local symmetry transformations can be written as

$$\psi(x) \rightarrow \exp(-iqf(x))\psi(x) \tag{2.8}$$

where $f(x)$ is an arbitrary real differentiable function of x.

2.1.2 Abelian gauge field theory

The electromagnetic interactions between matter (i.e., electrons) and massless gauge bosons (photon) are described by quantum electrodynamics (QED) corresponding to an Abelian $U(1)_{\text{e.m.}}$ group. One can start from the Lagrangian for the free Dirac field

$$\mathcal{L}_{\mathbf{0}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \tag{2.9}$$

where ψ is the free Dirac field and $\bar{\psi}$ is the Dirac conjugate, with the 4×4 matrices γ^μ, $\mu = 0, 1, 2, 3$ satisfying the anti-commutation relations

$$[\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu}, \quad \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0, \quad \gamma^{0\dagger} = \gamma^0 \tag{2.10}$$

$g^{\mu\nu}$ is the metric tensor, and γ^μ is defined as

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & \tau_1 \\ -\tau_1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \tau_2 \\ -\tau_2 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & \tau_3 \\ -\tau_3 & 0 \end{pmatrix} \tag{2.11}$$

The invariance of the Lagrangian (2.9) under global gauge transformation (2.7) allows us to change the phase of the field by the same amount at each space point [32]. To transform the Lagrangian (2.9) under the more general local symmetry (2.8), one gets an extra term $q\bar{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x)$ from the partial derivative and the Lagrangian is no longer invariant, and can be written as:

$$\mathcal{L} = \mathcal{L}_\mathbf{0} - q\bar{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x) \tag{2.12}$$

The new term can be compensated by introducing the vector field A_μ to have the transformation property

$$A_\mu \to A_\mu + \partial_\mu f(x) \tag{2.13}$$

The interaction between matter and gauge field can be obtained through the minimal substitution (replace partial derivative by covariant derivative)

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu \tag{2.14}$$

and

$$D_\mu\psi(x) = [\partial_\mu + iqA_\mu]\psi(x) \tag{2.15}$$

Now the Lagrangian is invariant under the gauge transformation after substitution from Eqs. (2.13) and (2.15) into Eq. (2.12). One should include the gauge invariant term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ for the vector field A_μ, where the field strength is defined as [52]:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.16}$$

The new Lagrangian is known as the Lagrangian of QED which is invariant under $U(1)$ transformation and can be rewritten as:

$$\mathcal{L}^{\mathrm{QED}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.17}$$

This gauge invariant Lagrangian describes the interaction of a massless vector field with a spinor field.

2.1.3 Non-Abelian gauge field theory

The Yang-Mills fields can be described by a special unitary group of degree n denoted $SU(n)$, which has $n \times n$ unitary matrices with unit determinant. The $SU(n)$ group is a continuous, internal symmetry group, and non-Abelian, where the elements of the group do not commute with each other. $SU(n)$ contains more interesting subgroups in particle physics such as $SU(3)$ which describes the strong interactions between quarks and $SU(2)$ which describes the weak interactions [31]. An element of $SU(2)$ can be parameterized by the three Pauli spin matrices [49]:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.18}$$

where the τ are 2×2 matrices which satisfy the commutation relations

$$[\tau_i, \tau_j] = 2i\varepsilon_{ijk}\tau_k \tag{2.19}$$

and give a representation of the Lie algebra with the antisymmetric structure constant ε_{ijk} tensor.

The leptonic currents and consequently the leptonic interactions involve only the left-handed lepton fields, so one can write the Dirac fields as:

$$\begin{aligned}
\psi^L(x) &= P_L\psi(x) = \frac{1}{2}(1-\gamma_5)\psi(x) \\
\psi^R(x) &= P_R\psi(x) = \frac{1}{2}(1+\gamma_5)\psi(x) \\
\bar{\psi}^L(x) &= \bar{\psi}(x)P_R = \bar{\psi}(x)\frac{1}{2}(1+\gamma_5) \\
\bar{\psi}^R(x) &= \bar{\psi}(x)P_L = \bar{\psi}(x)\frac{1}{2}(1-\gamma_5)
\end{aligned} \tag{2.20}$$

with $P_L^2 = P_L$, $P_R^2 = P_R$, $P_L + P_R = 1$, and γ_5 is defined as

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.21}$$

and has the properties

$$[\gamma^\mu, \gamma^5]_+ = 0, \quad (\gamma^5)^2 = 1, \quad \gamma^{5\dagger} = \gamma^5. \tag{2.22}$$

The lepton isospin doublet can be written in the form

$$\Psi_l^L(x) = \begin{pmatrix} \psi_{\nu_l}^L \\ \psi_l^L \end{pmatrix}, \tag{2.23}$$

and its Dirac conjugated can be written as

$$\bar{\Psi}_l^L(x) = \begin{pmatrix} \bar{\psi}_{\nu_l}^L & \bar{\psi}_l^L \end{pmatrix} \tag{2.24}$$

with $\bar{\psi}(x)$ defined by

$$\bar{\psi}(x) = \psi^\dagger(x)\gamma^0 \tag{2.25}$$

The free-lepton Lagrangian density in terms of left and right-handed fields takes the form

$$\mathcal{L}_{\mathbf{0}} = i\left[\bar{\Psi}_l^L\gamma^\mu\partial_\mu\Psi_l^L + \bar{\psi}_l^R\gamma^\mu\partial_\mu\psi_l^R + \bar{\psi}_{\nu_l}^R\gamma^\mu\partial_\mu\psi_{\nu_l}^R\right] \tag{2.26}$$

which is invariant under the global $SU(2)$ transformation of the left and right-handed fields

$$\begin{aligned}
\Psi_l^L(x) \to \Psi_l'^L(x) &= U(\alpha)\Psi_l^L(x) \equiv \exp\left(i\alpha_j\tau_j/2\right)\Psi_l^L(x) \\
\bar{\Psi}_l^L(x) \to \bar{\Psi}_l'^L(x) &= \bar{\Psi}_l^L(x)U^\dagger(\alpha) \equiv \bar{\Psi}_l^L(x)\exp\left(-i\alpha_j\tau_j/2\right) \\
\psi_l^R(x) \to \psi_l'^R(x) &= \psi_l^R(x), \quad \psi_{\nu_l}^R(x) \to \psi_{\nu_l}'^R(x) = \psi_{\nu_l}^R(x) \\
\bar{\psi}_l^R(x) \to \bar{\psi}_l'^R(x) &= \bar{\psi}_l^R(x), \quad \bar{\psi}_{\nu_l}^R(x) \to \bar{\psi}_{\nu_l}'^R(x) = \bar{\psi}_{\nu_l}^R(x)
\end{aligned} \tag{2.27}$$

The operators $U(\alpha)$ are 2×2 unitary matrices with the property $\mathbf{detU}(\alpha) = \mathbf{+1}$ for $SU(2)$, and $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ are real numbers. The $SU(2)$ transformation properties of the two-component left-handed lepton fields $\Psi_l^L(x)$ are the same as for the two-component spinors which describe spin $\frac{1}{2}$ particles in the non-relativistic Pauli theory of spin, so $\Psi_l^L(x)$ is called an isospinor [32].

For infinitesimal α_j, the transformations (2.27) can be reduced to

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l'^L(x) &= (1 + i\alpha_j\tau_j/2)\Psi_l^L(x) \\ \bar{\Psi}_l^L(x) \to \Psi_l'^L(x) &= \bar{\Psi}_l^L(x)(1 - i\alpha_j\tau_j/2) \end{aligned} \tag{2.28}$$

To generalize the transformation of the Lagrangian, one can replace the global transformation by a local one, which is given by

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l'^L(x) &= \exp\left(ig\tau_j\omega_j(x)/2\right)\Psi_l^L(x) \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l'^L(x) &= \bar{\Psi}_l^L(x)\exp\left(-ig\tau_j\omega_j(x)/2\right) \\ \psi_l^R(x) \to \psi_l'^R(x) &= \psi_l^R(x), \quad \psi_{\nu_l}^R(x) \to \psi_{\nu_l}'^R(x) = \psi_{\nu_l}^R(x) \\ \bar{\psi}_l^R(x) \to \bar{\psi}_l'^R(x) &= \bar{\psi}_l^R(x), \quad \bar{\psi}_{\nu_l}^R(x) \to \bar{\psi}_{\nu_l}'^R(x) = \bar{\psi}_{\nu_l}^R(x) \end{aligned} \tag{2.29}$$

where g is the coupling constant, and $\omega_j(x)$, $j = 1, 2, 3$ are three differentiable functions of x. The Lagrangian (2.26) is not invariant under the local transformation (2.29). For infinitesimal transformations it transforms as

$$\mathcal{L}_\mathbf{0} \to \mathcal{L}_\mathbf{0}' \equiv \mathcal{L}_\mathbf{0} - \frac{1}{2}g\bar{\Psi}_l^L(x)\tau_j\gamma^\mu\partial_\mu\omega_j(x)\Psi_l^L(x) \tag{2.30}$$

To obtain invariance of the Lagrangian (2.30), one has to replace the partial derivative by the covariant one, and to introduce the coupling of leptons to the gauge fields as follows

$$\partial^\mu\Psi_l^L(x) \to D^\mu\Psi_l^L(x) = [\partial^\mu + ig\tau_j W_j^\mu(x)/2]\Psi_l^L(x) \tag{2.31}$$

with

$$D^\mu\Psi_l^L(x) \to \exp\left(ig\tau_j\omega_j(x)/2\right)D^\mu\Psi_l^L(x) \tag{2.32}$$

and

$$W_j^\mu(x) \to W_j'^\mu(x) = W_j^\mu(x) + \delta W_j^\mu(x). \tag{2.33}$$

For infinitesimal transformations $\omega_j(x)$, the transformation (2.29) can be rewritten as

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l'^L(x) &= \left(1 + ig\tau_j\omega_j(x)\right)\Psi_l^L(x) \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l'^L(x) &= \bar{\Psi}_l^L(x)\left(1 - ig\tau_j\omega_j(x)\right) \end{aligned} \tag{2.34}$$

By neglecting the second-order terms and using Eqs. (2.31), (2.32), and (2.34) one can rewrite Eq. (2.33) as

$$W_i^\mu(x) \to W_i'^\mu(x) = W_i^\mu(x) - \partial^\mu\omega_i(x) - g\varepsilon_{ijk}\omega_j(x)W_k^\mu(x) \tag{2.35}$$

After substitution of Eqs. (2.31), (2.32), (2.35) into Eq. (2.30), one can get the invariant Lagrangian.

The global and local gauge symmetry $SU_L(2) \times U_Y(1)$ can be introduced in terms of the hypercharge Y, the isospin I_3^W and the electric charge Q through the Gell-Mann Nishijima relation [53]

$$Y = Q/e - I_3^W \tag{2.36}$$

where $Y = -\frac{1}{2}$, or $-\frac{1}{6}$ for left-handed isodoublet leptons and quarks, respectively and $Y = -1$ for singlet right-handed leptons, or $\frac{2}{3}$ and $-\frac{1}{3}$ for singlet right-handed up-quarks and down-quarks, respectively.

The Lagrangian density describing the electromagnetic and the weak interactions of leptons and gauge bosons is obtained from the following symmetries: The global transformation $U_Y(1)$ is

$$\begin{aligned} \Psi(x) \to \Psi'(x) &= \exp(i\beta Y)\Psi(x) \\ \bar{\Psi}(x) \to \bar{\Psi}'(x) &= \bar{\Psi}\exp(-i\beta Y) \end{aligned} \tag{2.37}$$

and the local transformation for $U_Y(1)$ is

$$\begin{aligned} \Psi(x) \to \Psi'(x) &= \exp(ig'Yf(x))\Psi(x) \\ \bar{\Psi}(x) \to \bar{\Psi}'(x) &= \exp(-ig'Yf(x))\bar{\Psi}(x) \end{aligned} \tag{2.38}$$

Therefore the leptonic Lagrangian density (2.26) is invariant under global transformations (2.37), and also under local transformations (2.38) if one replaces the partial derivative by the covariant one as follows

$$\partial_\mu \Psi(x) \to D_\mu \Psi(x) = [\partial_\mu + ig'YB_\mu(x)]\Psi(x) \tag{2.39}$$

where the fields $B^\mu(x)$ of the hypercharge are transformed as

$$B^\mu(x) \to B'^\mu(x) = B^\mu(x) - \partial^\mu f(x) \tag{2.40}$$

There is another example of a non-Abelian field which is described by the $SU(3)$ group, i.e, QCD is the theory describing the interactions between quarks (fermions with three colors referred to as red, blue and green), the quarks are bound by the gluons. The QCD Lagrangian for a given free quark flavor (u,c,t,d,s,b) takes the form

$$\mathcal{L}_0^{\text{QCD}} = \bar{q}_i(x)[i\gamma^\mu \partial_\mu - m]q_i(x) \tag{2.41}$$

where the sum over colors, $i = 1, 2, 3$. The Lagrangian (2.41) is not invariant under the local transformations

$$\begin{aligned} q(x) &\to \exp(i\alpha_a(x)T_a)q(x) \\ D_\mu q(x) &= (\partial_\mu + igT_aA_\mu)q(x) \end{aligned} \tag{2.42}$$

where T_a is the generator of the $SU(3)$ group, and satisfies the commutation relations

$$[T_a, T_b] = if_{abc}T_c \tag{2.43}$$

$\alpha_a(x)$ are eight functions, and f_{abc} is the structure constant of $SU(3)$.

To achieve the invariance of the Lagrangian under local transformations, it is necessary to replace the partial derivative by the covariant one, and use the QCD field-tensor as

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu \tag{2.44}$$

where A^a_μ are 8 gluon fields, and g is the QCD coupling.

The complete gauge invariant QCD Lagrangian can be written as

$$\mathcal{L}^{\text{QCD}} = \bar{q}_i(x)[i\gamma^\mu \partial_\mu - m]q_i(x) - g(\bar{q}\gamma^\mu T_a q)A^a_\mu - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \tag{2.45}$$

In the next section, we introduce the unification theory of the weak interaction $SU(2)$ and the electromagnetic theory $U(1)$ (QED) through the electroweak theory $SU(2)\times U(1)$.

2.2 Electroweak Theory

The electroweak sector of the standard model is described by the $SU(2) \times U(1)$ gauge theory. The leptonic Lagrangian density $\mathcal{L}^L$ (the superscript L of the Lagrangian here refers to "leptons") is gauge-invariant under the local $SU_L(2) \times U_Y(1)$ gauge symmetry group (apply local $SU_L(2)$ transformations of Eq. (2.29), and local $U_Y(1)$ transformations Eq. (2.38)), you will have

$$\mathcal{L}^L = i[\bar{\Psi}_l^L \gamma^\mu D_\mu \Psi_l^L + \bar{\psi}_l^R \gamma^\mu D_\mu \psi_l^R + \bar{\psi}_{\nu_l}^R \gamma^\mu D_\mu \psi_{\nu_l}^R] \tag{2.46}$$

where

$$\begin{aligned} D^\mu \Psi_l^L(x) &= [\partial^\mu + ig\tau_j W_j^\mu(x)/2 - ig' B^\mu(x)/2] \Psi_l^L(x) \\ D^\mu \psi_l^R(x) &= [\partial^\mu - ig' B^\mu(x)/2] \psi_l^R(x) \\ D^\mu \psi_{\nu_l}^R(x) &= \partial^\mu \psi_{\nu_l}^R(x) \end{aligned} \tag{2.47}$$

where g is the coupling constant corresponding to $SU(2)_L$, and g' is the coupling constant corresponding to $U(1)_Y$.

The fields $W_i^\mu(x)$ are invariant under $U(1)$ gauge transformations, and the $B^\mu(x)$ fields are invariant under $SU(2)$ gauge transformations. The non-hermitian charged gauge fields are defined as

$$\begin{aligned} W_\mu(x) &= \frac{1}{\sqrt{2}}[W_{1\mu}(x) - iW_{2\mu}(x)] \\ W_\mu^\dagger(x) &= \frac{1}{\sqrt{2}}[W_{1\mu}(x) + iW_{2\mu}(x)] \end{aligned} \tag{2.48}$$

The hermitian neutral fields are defined as linear combinations of the $A_\mu(x)$ and Z_μ(x) fields as follows

$$\begin{aligned} W_{3\mu}(x) &= \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x) \\ B_\mu(x) &= -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x) \end{aligned} \tag{2.49}$$

where θ_W is the Weinberg angle and also is defined by $\tan\theta_W = g'/g$. It is important to include the gauge bosons to the Lagrangian Eq. (2.46), and this can be done by introducing the $U(1)$ gauge-invariant Lagrangian density of the fields $B^\mu(x)$ as follows

$$\mathcal{L}_G^{U_Y(1)} = -\frac{1}{4} B_{\mu\nu}(x) B^{\mu\nu}(x) \tag{2.50}$$

where

$$B^{\mu\nu}(x) = \partial^\nu B^\mu(x) - \partial^\mu B^\nu(x) \tag{2.51}$$

Also by introducing the $SU(2)$ gauge-invariant Lagrangian density of the gauge boson fields $W^\mu(x)$

$$\mathcal{L}_G^{SU(2)} = -\frac{1}{4} G_{i\mu\nu}(x) G^{i\mu\nu}(x) \tag{2.52}$$

where

$$
\begin{aligned}
G_i^{\mu\nu}(x) &= F_i^{\mu\nu}(x) + g\varepsilon_{ijk}W_j^{\mu}(x)W_k^{\nu}(x) \\
F_i^{\mu\nu}(x) &= \partial^{\nu}W_i^{\mu}(x) - \partial^{\mu}W_i^{\nu}(x)
\end{aligned}
\tag{2.53}
$$

The $SU(2)\times U(1)$ gauge-invariant Lagrangian density of the gauge boson fields is the sum of Eqs. (2.50) and (2.52) which takes the form

$$
\begin{aligned}
\mathcal{L}^B &= -\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x) \\
&= -\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \frac{1}{4}F_{i\mu\nu}(x)F_i^{\mu\nu}(x) \\
&\quad + g\varepsilon_{ijk}W_{i\mu}(x)W_{j\nu}(x)\partial^{\mu}W_k^{\nu}(x) \\
&\quad - \frac{1}{4}g^2\varepsilon_{ijk}\varepsilon_{ilm}W_j^{\mu}(x)W_k^{\nu}(x)W_{l\mu}(x)W_{m\nu}(x)
\end{aligned}
\tag{2.54}
$$

and can be rewritten into two parts as

$$
\mathcal{L}^B = \mathcal{L}_{\mathbf{0}}^B + \mathcal{L}_{\mathbf{I}}^B \tag{2.55}
$$

The first line of the Lagrangian (2.54) represents the free-field Lagrangian density of massless, spin 1 gauge bosons γ, $W^{\pm}$ and Z^0 and takes the form

$$
\mathcal{L}_{\mathbf{0}}^B = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - \frac{1}{2}F_{W\mu\nu}^{\dagger}(x)F_W^{\mu\nu}(x) - \frac{1}{4}Z_{\mu\nu}(x)Z^{\mu\nu}(x) \tag{2.56}
$$

where

$$
Z^{\mu\nu}(x) = \partial^{\nu}Z^{\mu}(x) - \partial^{\mu}Z^{\nu}(x), \tag{2.57}
$$

$F_{\mu\nu}(x)$ is the electromagnetic tensor and $F_W^{\mu\nu}(x)$ is corresponding to the definition of the tensor in Eq. (2.53).

The second and third lines of the Lagrangian (2.54) represent the Lagrangian density for the interactions of fields

$$
\begin{aligned}
\mathcal{L}_{\mathbf{I}}^B &= g\varepsilon_{ijk}W_{i\mu}(x)W_{j\nu}(x)\partial^{\mu}W_k^{\nu}(x) \\
&\quad - \frac{1}{4}g^2\varepsilon_{ijk}\varepsilon_{ilm}W_j^{\mu}(x)W_k^{\nu}(x)W_{l\mu}(x)W_{m\nu}(x)
\end{aligned}
\tag{2.58}
$$

The unified model of electromagnetic and weak interactions of massless leptons and massless gauge bosons ($W^{\pm}$, Z^0 bosons and photons) can be described by the Lagrangian density (the sum of Eqs. (2.46) and (2.54)) as

$$
\mathcal{L} = \mathcal{L}^L + \mathcal{L}^B \tag{2.59}
$$

To give masses to the gauge bosons $W^{\pm}$ and $Z^{\mathbf{0}}$, one might want to add a mass term to the Lagrangian (2.56), such as:

$$
\textbf{mass term} = m_W^2 W_{\mu}^{\dagger}(x)W^{\mu}(x) + \frac{1}{2}m_Z^2 Z_{\mu}(x)Z^{\mu}(x) \tag{2.60}
$$

where $m_W = vg/2$ and $m_Z = m_W cos\theta_W$. For fermions, one might also add a mass term to the Lagrangian (2.46) such as:

$$\mathbf{M}\bar{\Psi}\Psi = \mathbf{M}[\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L] \tag{2.61}$$

However, such mass terms are not invariant under local and global symmetry due to the different gauge transformations of the left and right-handed fields. A Lagrangian density including such mass terms is not invariant under $SU(2)$ and $U(1)$.

Spontaneous electroweak symmetry breaking is the good way to give the $W^{\pm}$, Z bosons and fermions masses while keeping the gauge theory $SU(2) \times U(1)$ invariant.

All fields we discussed are massless, we will in the next section discuss how to introduce masses to the gauge bosons $W^{\pm}$ and Z of the weak interactions and the fermions.

2.2.1 Spontaneous symmetry breaking

In this section one tries to generalize the unification of the electromagnetic and weak theory $SU(2) \times U(1)$ by constructing a renormalized and gauge-invariant Lagrangian density, which contains mass terms for fermions and for the $W^{\pm}$ and Z bosons, while the photons remain massless. The procedure responsible for giving mass to bosons and fermions is called the Higgs mechanism due to the spontaneous breaking down of the electroweak symmetry [54]. Spontaneous symmetry breaking takes place when a system or Lagrangian density that is symmetric with respect to some symmetry groups goes into a vacuum state that is not symmetric or not invariant.

The Goldstone model is one example of spontaneous symmetry breaking and its Lagrangian density is defined as

$$\mathcal{L}(x) = [\partial^{\mu}\phi^{\star}(x)][\partial_{\mu}\phi(x)] - V(\phi) \tag{2.62}$$

where the complex scalar field can be written as:

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + i\phi_2(x)] \tag{2.63}$$

The Higgs potential takes the form

$$V(\phi) = \mu^2|\phi(x)|^2 + \lambda|\phi(x)|^4 \tag{2.64}$$

where λ and μ^2 are real parameters, λ must be positive to make the potential bounded from below.

There are two situations to minimize the potential $V(\phi)$ according to the sign of μ^2:

- $\mu^2 > 0$, the potential $V(\phi)$ is positive and has a unique minimum point value at $\langle 0|\phi(x)|0\rangle \equiv \phi_0 = 0$, i.e., spontaneous symmetry breaking cannot occur, this case is shown in Fig 2.1.

- $\mu^2 < 0$, the potential $V(\phi)$ has a minimum when $\frac{\partial V}{\partial \phi} = 2\mu^2\phi + 4\lambda\phi^3 = 0$, i.e., $V(\phi)$ has a local maximum at $\phi(x) = 0$ and a whole circle of minima at

$$\langle 0|\phi(x)|0\rangle \equiv \phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} = \frac{v}{\sqrt{2}} \tag{2.65}$$

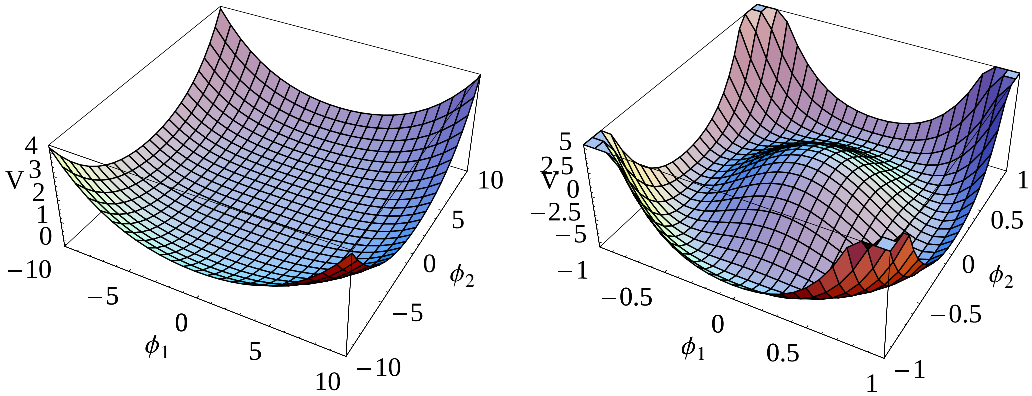

Figure 2.1: Higgs potential

which corresponds to a continuum of vacuum states degenerate in energy, therefore the ground state is not unique as shown in Fig. 2.1.

To study the deviation of fields from the vacuum expectation value (excited states), one introduces two real Klein-Gordon fields $\sigma(x)$ and $\eta(x)$ to expand the scalar fields around the physical vacuum

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x) + i\eta(x)] \tag{2.66}$$

By substitution from Eq. (2.66) into Eq. (2.62), the new Lagrangian can be rewritten in terms of the $\sigma(x)$ and $\eta(x)$ fields as

$$\begin{aligned}\mathcal{L}(x) \;\; = \;\; & \frac{1}{2}[\partial^\mu \sigma(x)][\partial_\mu \sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) \\ & + \frac{1}{2}[\partial^\mu \eta(x)][\partial_\mu \eta(x)] \\ & - \lambda v \sigma(x)[\sigma^2(x) + \eta^2(x)] - \frac{1}{4}\lambda[\sigma^2(x) + \eta^2(x)]^2\end{aligned} \tag{2.67}$$

The higher order terms are considered as interaction terms. By taking out the interaction terms, Eq. (2.67) describes two particles, the first one is the σ boson with spin 0 and mass $\sqrt{2\lambda v^2}$, and the other one is the η boson without mass and called a Goldstone boson.

2.2.2 Higgs mechanism

The Higgs mechanism is an extension of the spontaneous symmetry breaking to create massive vector bosons in a gauge invariant theory [54]. The simple case of the Abelian $U(1)$ field can be obtained by introducing the coupling of the gauge field through the covariant derivative

$$D_\mu \phi(x) \;\; = \;\; [\partial_\mu + iqA_\mu(x)]\phi(x) \tag{2.68}$$

and by adding the free gauge field to the Lagrangian density (2.62)

$$\begin{aligned}& -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x), \\ & F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x).\end{aligned} \tag{2.69}$$

This can be rewritten as

$$\begin{aligned}\mathcal{L}(x) &= [D^\mu\phi(x)]^\star[D_\mu\phi(x)] - \mu^2|\phi(x)|^2 - \lambda|\phi(x)|^4 \\ &\quad -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)\end{aligned} \tag{2.70}$$

which defines the Higgs model and is invariant under a $U(1)$ local gauge transformation

$$\begin{aligned}\phi(x) \to \phi'(x) &= \phi(x)\exp\left(-iqf(x)\right) \\ \phi^\star(x) \to \phi'^\star(x) &= \phi^\star(x)\exp\left(iqf(x)\right) \\ A_\mu(x) \to A'_\mu(x) &= A_\mu(x) + \partial f(x)\end{aligned} \tag{2.71}$$

For $\mu^2 > 0$, $\mathcal{L}$ is simply the QED-like Lagrangian for a charged scalar particle of mass μ and with ϕ^4 self-interactions. For $\mu^2 < 0$, the vacuum state is not unique and spontaneous symmetry breaking will occur. Therefore the Lagrangian can be written in terms of real Klein-Gordon fields and a gauge field $A^\mu(x)$ as follows:

$$\begin{aligned}\mathcal{L}(x) &= [\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) \\ &\quad -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}(qv)^2A_\mu(x)A^\mu(x) \\ &\quad +[\partial^\mu\eta(x)][\partial_\mu\eta(x)] \\ &\quad +qvA^\mu(x)\partial_\mu\eta(x) + \text{interaction terms}\end{aligned} \tag{2.72}$$

There are two difficulties to understand Eq. (2.72). The first problem is that there is no physical interpretation for the mixing term of the A^μ and η fields, the second problem is the number of degrees of freedom. Before symmetry breaking in Eq. (2.70) there are four (two for the complex scalar field $\phi(x)$ and two for the massless gauge field A^μ). After symmetry breaking in Eq. (2.72) there appear to be five degrees of freedom (two for the real scalar fields σ and η and three for the massive gauge boson field A^μ). To solve these problems the unphysical field η should be removed from Eq. (2.72) under unitary gauge.

The scalar field $\phi(x)$ can be rewritten as

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x)] \tag{2.73}$$

The free-field Lagrangian density of Eq. (2.72) in unitary gauge then takes the form

$$\begin{aligned}\mathcal{L}_0(x) &= [\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) \\ &\quad -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}(qv)^2A_\mu(x)A^\mu(x)\end{aligned} \tag{2.74}$$

This equation represents a massive gauge field A^μ with mass qv and a scalar field σ-boson with mass $\sqrt{2\lambda v^2}$ which is called the Higgs boson.

One can extend the Higgs mechanism to the $SU(2)\times U(1)$ gauge invariant Lagrangian of the electroweak theory or Weinberg-Salam Model by replacing the singlet scalar field

$\phi(x)$ in the $U(1)$ group by the scalar isospin doublet $\Phi(x)$ in the $SU(2)$ group, which can be written around the vacuum state as

$$\Phi(x) = \begin{pmatrix} \phi_a(x) \\ \phi_b(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ v + \sigma(x) + i\eta_3(x) \end{pmatrix} \tag{2.75}$$

The isospin doublet transforms under local $SU(2)$ transformations according to

$$\begin{aligned} \Phi(x) \rightarrow \Phi'(x) &= \exp\left(ig\tau_j\omega_j(x)/2\right)\Phi(x) \\ \Phi^\dagger(x) \rightarrow \Phi^{\dagger\prime}(x) &= \Phi^\dagger(x)\exp(-ig\tau_j\omega_j(x)/2) \end{aligned} \tag{2.76}$$

and under local $U(1)$ weak hypercharge transformations as

$$\begin{aligned} \Phi(x) \rightarrow \Phi'(x) &= \exp\left(ig'Yf(x)\right)\Phi(x) \\ \Phi^\dagger(x) \rightarrow \Phi^{\dagger\prime}(x) &= \Phi^\dagger(x)\exp\left(-ig'Yf(x)\right) \end{aligned} \tag{2.77}$$

To generalize the Lagrangian density Eq. (2.59) of interactions between lepton and gauge bosons, one can include the Higgs field $\Phi(x)$, therefore the Lagrangian density takes the form

$$\mathcal{L} = \mathcal{L}^F + \mathcal{L}^B + \mathcal{L}^H \tag{2.78}$$

where

$$\mathcal{L}^H(x) = [D^\mu\Phi(x)]^\dagger[D_\mu\Phi(x)] - \mu^2\Phi^\dagger(x)\Phi(x) - \lambda[\Phi^\dagger(x)\Phi(x)]^2 \tag{2.79}$$

with

$$D^\mu\Phi(x) = [\partial^\mu + ig\tau_j W_j^\mu(x)/2 + ig'YB^\mu(x)]\Phi(x) \tag{2.80}$$

and the fermion Lagrangian $\mathcal{L}^F$ can be defined as

$$\mathcal{L}^F = \sum_{F=u,d,l} \bar{F}iD_\mu\gamma^\mu F \tag{2.81}$$

For $\mu^2 < 0$ and $\lambda > 0$, the vacuum expectation values (v.e.v.'s) for the Higgs fields with hypercharge $Y = \frac{1}{2}$ and isospin $I_3^W = -\frac{1}{2}$ can be written as

$$\Phi(x) = \begin{pmatrix} \phi_a^0(x) \\ \phi_b^0(x) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{2.82}$$

The ground state (2.82) is not invariant under $SU(2) \times U(1)$ gauge transformations (the lower component of the Higgs doublet or neutral component is not equal zero), but it must be invariant under $U(1)$ transformations to ensure that the conservation of charge is valid and the photons are massless.

After substitution from Eqs. (2.75) and (2.80) into Eq. (2.79), the symmetry is spontaneously broken down and the degrees of freedom of the η-fields are "eaten" by the massive gauge bosons $W^\pm$ and Z^0. The real magic of the Higgs mechanism is how to give mass to the $W^{\pm}$ and Z^0-bosons, leaving the photons massless. The v.e.v. v of the Higgs field gives the $W^{\pm}$ a mass (tree-level) $\frac{vg}{2}$, and the Z^0 a mass (tree-level) $\frac{v}{2}\sqrt{g^2+g'^2}$.

One can count the number of degrees of freedom before and after symmetry breaking as: before symmetry breaking there are twelve degrees of freedom, four degrees for the complex doublet (three degrees from the scalar η-fields and one from the scalar σ-field) and eight degrees from the four massless gauge bosons. After symmetry breaking there are also twelve degrees of freedom, nine from the three massive gauge bosons $W^{\pm}$, and Z^0 (the three degrees of η-field are eaten by the longitudinal polarization components of $W^{\pm}$ and Z^0 bosons) and two from the massless gauge bosons (photons) and one for the massive scalar field σ (Higgs boson).

In the weak basis the Yukawa interactions of the quarks with the $SU(2)$ doublet Higgs field are described by two 3×3 coupling matrices. The way to give masses for fermions, is by Yukawa interactions between the Higgs fields and the fermion fields, which is defined as

$$-\mathcal{L}^Y(x) = \sum_{\text{generations}} \left[\bar{\Psi}^L(x) g^d \Phi(x)\psi_d^R + \bar{\Psi}^L(x) g^u \tilde{\Phi}(x)\psi_u^R + \bar{\Psi}_l^L(x) g^e \Phi(x)\psi_l^R\right] + \text{h.c.} \tag{2.83}$$

and is invariant under $SU(2)\times U(1)$, where $\tilde{\Phi}(x)$ is defined as

$$\tilde{\Phi}(x) = -i[\Phi^{\dagger}(x)\tau_2]^T = \begin{pmatrix} \phi_b^{\star}(x) \\ -\phi_a^{\star}(x) \end{pmatrix} \tag{2.84}$$

and transformed under $U(1)$ as

$$\tilde{\Phi}(x) \to \tilde{\Phi}'(x) = \exp\left(-ig' f(x)/2\right)\tilde{\Phi}(x) \tag{2.85}$$

After transforming the quark fields from the weak basis to the mass basis, various pieces of the SM Lagrangian are diagonal in generation space (unitary gauge), except for the charged current interactions of quarks,

$$-\mathcal{L}^Y_{cc}(x) = -\frac{g}{\sqrt{2}}\bar{\Psi}^L_u(x)\gamma^{\mu}V_{\text{CKM}}\Psi^L_d W^+_{\mu} + \text{h.c.} \tag{2.86}$$

Therefore Eq. (2.78) can be generalized to

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}^F + \mathcal{L}^B + \mathcal{L}^H + \mathcal{L}^Y \\
&= i\left[\bar{\Psi}_l^L(x)\gamma^{\mu}D_{\mu}\Psi_l^L(x) + \bar{\psi}_l^R(x)\gamma^{\mu}D_{\mu}\psi_l^R(x) + \bar{\psi}_{\nu_l}^R(x)\gamma^{\mu}D_{\mu}\psi_{\nu_l}^R(x)\right] \\
&- \frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x) \\
&+ [D^{\mu}\Phi(x)]^{\dagger}[D_{\mu}\Phi(x)] - \mu^2\Phi^{\dagger}(x)\Phi(x) - \lambda[\Phi^{\dagger}(x)\Phi(x)]^2 \\
&+ \Big(\sum_{generations}\left[\bar{\Psi}^L(x) g^d \Phi(x)\psi_d^R + \bar{\Psi}^L(x) g^u \tilde{\Phi}(x)\psi_u^R + \bar{\Psi}_l^L(x) g^e \Phi(x)\psi_l^R\right] + \text{h.c.}\Big)
\end{aligned} \tag{2.87}$$

where the covariant derivative for left and right-handed lepton fields is defined by Eq. (2.47) and for scalar fields (Higgs fields), it is defined by Eq. (2.80).

In the next section, we try to discuss the theoretical and experimental constraints on the mass m_H of the Higgs boson because the lightest neutral Higgs in the 2HDM is analogous to the SM Higgs boson.

2.3 Limits on the Standard Model Higgs mass

The Standard Model of the electroweak physics is in good agreement with data, the only particle of the Standard Model that has not been detected so far is the Higgs boson [27]. The precision experiments put limits on the Higgs mass through its influence in radiative corrections (higher-order terms of the perturbative series) [55]. The Tevatron at Fermilab provides an indirect upper limit on the Higgs mass of 219 GeV via precision measurements of m_t and m_W. Direct searches at LEP give a lower bound of $m_H = 114.4$ GeV [56]. Therefore the preferred mass range of the Higgs boson in the SM is 114 GeV$< m_H <$ 219 GeV at the 95% Confidence Level (CL) [57]. Improved measurements of m_t and m_W at the Tevatron, and then at the LHC, will improve the precision of the indirect estimation of m_H.

In addition to these experimental limits there are theoretical constraints such as triviality, unitarity and vacuum stability bounds. The mass of the Higgs boson in the SM is a free parameter because it depends on the scalar self-coupling λ_R (renormalized self-coupling of the bare coupling λ), which is a free parameter. But λ_R should be perturbative up to a large scale, therefore this condition puts an upper bound on the Higgs mass, this is known as the triviality bound. Also the vacuum of the SM should be stable up to that large scale, this condition puts a lower bound on the Higgs mass.

In the SM of electroweak interactions the scalar potential is defined by

$$V(\phi) = \mu^2|\phi(x)|^2 + \lambda|\phi(x)|^4 \tag{2.88}$$

where λ is called the bare coupling, and its renormalized self-coupling is λ_R which lies in a narrow range ($\lambda_R \to 0$). The coupling constant λ at some low energy renormalization scale μ is defined as

$$\frac{1}{\lambda(\mu)} - \frac{1}{\lambda(\Lambda)} = \frac{3}{2\pi^2}\ln\frac{\Lambda}{\mu} \tag{2.89}$$

where Λ is the cutoff scale. The Higgs mass is found to be [58]

$$M_H^2 = 2\lambda_R(m_H)v^2 \tag{2.90}$$

Therefore for a given cutoff, the mass M_H is also found to be bounded from above [59,60].

Although the SM theory is a good approximation to the physics of elementary particles and their interactions at the low energy scale of $\mathcal{O}(100$ GeV) and below, the SM theory breaks down at some energy scale called Λ below the Plank scale ($M_{PL} \simeq 10^{19}$ GeV). The SM degrees of freedom are no longer adequate for describing physics above Λ and new physics must be relevant. Therefore the SM is not a fundamental theory and it is considered as an effective theory [61].

One can list some of the problems which are not explained by the SM, such as:

- Neutrino oscillation. The mass for neutrinos is not predicted by the SM.
- Gravity. Gravity becomes relevant above the Plank scale, which is not included in the SM.

- Mass parameters. The mass parameters in the SM theory are free parameters, new physics beyond the SM may fix these free parameters.

- Hierarchy problem. The mass term of the Higgs particle is of the order of the weak scale $\sim 10^2$ GeV and acquires quadratically divergent quantum corrections. If the cutoff $\Lambda_{\text{cut-off}}$ of the SM is comparable to the Plank scale, the parameters of the theory need to be carefully fine-tuned (cancellation between the quadratic radiative corrections and the bare mass) to keep the Higgs mass at an acceptable value.

  $$m^2_{\text{physics}} = m^2_0 + g^2\Lambda^2_{\text{cut-off}} \tag{2.91}$$

  New physics may solve this problem beyond the electroweak scale.

- CP Violation. CP violation is incorporated in the SM through the complex phase introduced to the elements of the CKM matrix. This is not sufficient to explain the baryogenesis in the universe [5]. New physics is needed to produce enough CP violation to explain why the universe is composed of much more matter than antimatter.

- Strong CP violation. Most of the proposed solutions for this problem, require an enlargement of the Higgs sector. A generic CP violation in the strongly interacting sector would create the electric dipole moment of the neutron.

Chapter 3

The Two Higgs Doublet Model

The mechanism of electroweak symmetry breaking is the most important ingredient in the description of elementary particles physics. The SM incorporates the Higgs mechanism that breaks the electroweak symmetry spontaneously through a neutral scalar field with non-zero v.e.v. In the minimal version of this mechanism one scalar $SU(2)_L$ doublet is required, providing one physical particle (the Higgs particle) [1–4].

CP violation is one of the crucial ingredients necessary to generate the observed matter-antimatter asymmetry of the Universe: It is not possible to generate a baryon asymmetry of the observed size with the very small CP violation present in the SM [5] via the complex phase in the CKM matrix. New sources of CP violation in models beyond the SM can play an important role in the explanation of the observed size of this asymmetry.

The simplest extension of the minimal Standard Model is the Two-Higgs doublet Model (2HDM), which is formed by adding an extra complex scalar doublet to the SM. As a consequence there exists a variety of new sources of CP violation, which are required to explain the matter-antimatter asymmetry in the Universe (baryon asymmetry) [6]. Various motivations for adding a second Higgs doublet to the SM have been advocated in the literature [7–10]. The quantity $\rho = M_W/(M_Z \cos^2\theta_W) = 1$ like the SM at tree level [11,12], if both Higgs fields are weak isodoublets ($T = 1/2$) with hypercharge $Y = 1$. After the electroweak-breaking mechanism, three of the eight degrees of freedom of the two complex scalar doublets are absorbed by the $W^\pm$ and Z^0 bosons to be their longitudinal components. The remaining five degrees of freedom form five elementary Higgs particles. The physical spectrum of the 2HDM contains five Higgs bosons: two neutral scalars (CP-even) h (the lightest one corresponding to the SM Higgs) and H(the heaviest one), and the neutral pseudoscalar A (CP-odd) and two charged scalars $H^\pm$ in the case of CP-conserving sector [13]. In the most general CP-violating 2HDM, the physical Higgs fields are linear combinations of h, H and A. Neutron electric dipole moment (NEDM) is a consequence of CP violation and can arise through the exchange of the neutral Higgs bosons [14]. Also Weinberg proposed a gauge theory of CP nonconservation through the exchange of the charged Higgs bosons [15]. These particles could be detected directly at LHC or indirectly through their contributions as intermediate states in decay process.

A discrete symmetry is often introduced to avoid flavor-changing neutral currents (FCNC) in the 2HDM [62] at tree-level. The 2HDM can be classified according to the Higgs-fermion interactions into: In type-I models [63] only one of the Higgs fields couples to standard model fermions (quarks and charged leptons). In type-II models [13] one Higgs field Φ_2 couples to up-type quarks ($I_3 = 1/2$), and the other Higgs field Φ_1 couples to down-type quarks and charged leptons ($I_3 = -1/2$). In type-III models [64] both Higgs fields couple to all standard model fermions, therefore it allows FCNC at tree-level. This model exhibits tree-level flavor-changing neutral Higgs interactions, but only in the top quark sector which can have significant effects on the electron dipole moment, and on $D - \bar{D}$ mixing and on $B - \bar{B}$ mixing [65]. Type-IV models [66] allow the Higgs field Φ_2 to couple to up-type and down-type quarks and the other Higgs field to couple to charged leptons.

In the next sections, we try to discuss the $SU(2)\times U(1)$ symmetry breaking Lagrangian of the 2HDM.

3.1 2HDM Lagrangian

There are four types of 2HDM according to the coupling of Higgs fields Φ_1 and Φ_2 to the fermions (up-type, down-type quarks and leptons). To avoid the FCNC at the tree-level we consider the 2HDM type-II [13] in the whole thesis.

The two doublets are defined in $SU(2)$ as

$$\Phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix}, \quad i = 1, 2 \tag{3.1}$$

To break down the symmetry at tree level, the v.e.v.'s for the two doublets can be written as

$$\Phi_i^0 = \begin{pmatrix} 0 \\ \frac{v_i}{\sqrt{2}} \end{pmatrix}, \quad i = 1, 2 \tag{3.2}$$

where

$$v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad v_1^2 + v_2^2 = v^2 = (246\ \text{GeV})^2 \tag{3.3}$$

and an important free parameter of the 2HDM is the ratio of the two v.e.v.'s $\tan\beta = v_2/v_1$, $(0 \leq \beta \leq \pi/2)$ which makes sense only if there is a physical principle that distinguishes between Φ_1 and Φ_2. Such a principle is model-dependent. The angle β rotates the CP-odd and the charged scalars into their mass eigenstates.

A spontaneous electroweak symmetry breaking for 2HDM is described by the Lagrangian

$$\mathcal{L} = \mathcal{L}^F + \mathcal{L}^B + \mathcal{L}^H + \mathcal{L}^Y. \tag{3.4}$$

The first two terms of Eq. (3.4) are the same for the standard model and for the 2HDM (see Eq. (2.87)), but $\mathcal{L}^H$ and $\mathcal{L}^Y$ are different. $\mathcal{L}^Y$ it will be discussed in detail in the next section.

The $SU(2) \times U(1)$ invariant Higgs Lagrangian for a system of scalar fields Φ_1 and Φ_2 can be written as

$$\mathcal{L}^H = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) - V \tag{3.5}$$

where V is the potential of the 2HDM, we will discuss it in detail in the next sections for the cases of CP-conservation and CP-violation.

The covariant derivative D_μ containing the electroweak gauge fields for $Y = 1$ for both doublets, is defined as

$$D^\mu = \partial^\mu + \frac{ig}{2}\tau_j W_j^\mu(x) + \frac{ig'}{2}B^\mu(x) \tag{3.6}$$

Using Pauli matrices Eq. (2.18), together with (2.48) and (2.49), one finds

$$\frac{ig}{2}\sum_{j=1}^{3}\tau_j W_j^\mu = \frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & W^\mu \\ W^{\dagger\mu} & 0 \end{pmatrix} + \frac{ig}{2}\begin{pmatrix} c_W Z_\mu + s_W A_\mu & 0 \\ 0 & -c_W Z_\mu - s_W A_\mu \end{pmatrix} \tag{3.7}$$

Therefore from Eqs. (3.1), (3.6) and (3.7), the quantity $D^\mu\Phi(x)$ can be rewritten as

$$\begin{aligned} D^\mu\Phi_1 \;=\;& \partial^\mu \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1+\eta_1+i\chi_1) \end{pmatrix} \\ &+\frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & W^\mu \\ W^{\dagger\mu} & 0 \end{pmatrix}\begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1+\eta_1+i\chi_1) \end{pmatrix} \\ &+\frac{ig}{2}\begin{pmatrix} c_W Z^\mu + s_W A^\mu & 0 \\ 0 & -c_W Z^\mu - s_W A^\mu \end{pmatrix}\begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1+\eta_1+i\chi_1) \end{pmatrix} \\ &+\frac{ig}{2}\frac{s_W}{c_W}(-s_W Z^\mu + c_W A^\mu)\begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1+\eta_1+i\chi_1) \end{pmatrix} \end{aligned} \tag{3.8}$$

Eq. (3.8) can be rewritten as

$$D^\mu\Phi_1 = \begin{pmatrix} \partial^\mu\varphi_1^+ + \frac{ig}{2c_W}\left[(c_W^2 - s_W^2)Z^\mu + 2c_W s_W A^\mu\right]\varphi_1^+ + \frac{ig}{2}W^\mu(v_1+\eta_1+i\chi_1) \\ \frac{\partial^\mu}{\sqrt{2}}(\eta_1+i\chi_1) - \frac{ig}{2\sqrt{2}c_W}Z^\mu(v_1+\eta_1+i\chi_1) + \frac{ig}{\sqrt{2}}W^{\dagger\mu}\varphi_1^+ \end{pmatrix} \tag{3.9}$$

where $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$, therefore

$$\begin{aligned} (D_\mu\Phi_i)^\dagger(D^\mu\Phi_i) \;=\;& \Big[\partial_\mu\varphi_i^- - \frac{ig}{2c_W}\left[(c_W^2 - s_W^2)Z_\mu + 2c_W s_W A_\mu\right]\varphi_i^- - \frac{ig}{2}W_\mu^\dagger(v_i+\eta_i-i\chi_i)\Big] \\ &\times\Big[\partial^\mu\varphi_i^+ + \frac{ig}{2c_W}\left[(c_W^2 - s_W^2)Z^\mu + 2c_W s_W A^\mu\right]\varphi_i^+ + \frac{ig}{2}W^\mu(v_i+\eta_i+i\chi_i)\Big] \\ &+\Big[\frac{\partial_\mu}{\sqrt{2}}(\eta_i - i\chi_i) + \frac{ig}{2\sqrt{2}c_W}Z^\mu(v_i+\eta_i-i\chi_i) - \frac{ig}{\sqrt{2}}W_\mu\varphi_i^-\Big] \\ &\times\Big[\frac{\partial^\mu}{\sqrt{2}}(\eta_i + i\chi_i) - \frac{ig}{2\sqrt{2}c_W}Z^\mu(v_i+\eta_i+i\chi_i) + \frac{ig}{\sqrt{2}}W^{\dagger\mu}\varphi_i^+\Big], \end{aligned} \tag{3.10}$$

where $i = 1, 2$.

After a little calculation, one can extract the mass of the $W^\pm$ and Z^0 bosons as follows:

$$m_W^2(W_\mu^\dagger W^\mu) = \frac{g^2}{4}(W_\mu^\dagger W^\mu)(v_1^2 + v_2^2) = \frac{g^2 v^2}{4}(W_\mu^\dagger W^\mu) \tag{3.11}$$

Therefore $m_W = \frac{1}{2}gv$. Likewise:

$$m_Z^2(Z_\mu Z^\mu) = \frac{v^2}{4}(g^2 + g'^2)(Z_\mu Z^\mu) \tag{3.12}$$

and $m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}$.

The Higgs-vector-boson couplings can be extracted from the general form the covariant derivative terms, see Appendix B of Ref. [17].

3.2 Yukawa coupling

The Yukawa interaction is an interaction between a scalar field Φ and the Dirac field Ψ. Through spontaneous symmetry breaking, the fermions acquire a mass proportional to the v.e.v. of the Higgs field.

The Yukawa Lagrangian is a generalization of the similar form of the SM, and can be written in terms of the quark and lepton mass-eigenstate fields as [6, 67, 68]

$$\begin{aligned} -\mathcal{L}^Y &= \bar{\Psi}_q^L g_1^d \Phi_1 \psi_d^R + \bar{\Psi}_q^L g_1^u \tilde{\Phi}_1 \psi_u^R \\ &+ \bar{\Psi}_q^L g_2^d \Phi_2 \psi_d^R + \bar{\Psi}_q^L g_2^u \tilde{\Phi}_2 \psi_u^R \\ &+ \bar{\Psi}_l^L g_1^e \Phi_1 \psi_l^R + \bar{\Psi}_l^L g_2^e \Phi_2 \psi_l^R + h.c. \end{aligned} \tag{3.13}$$

with $g_i^F (i = 1, 2$ and F for fermions) the Yukawa interaction matrices, Ψ_q^L the left-handed quark fields and Ψ_l^L the left-handed lepton fields. The field $\tilde{\Phi}_i$ is defined in analogy with Eq. (2.84) as

$$\begin{aligned} \tilde{\Phi}_i &= -i[\Phi_i^\dagger \tau_2]^T = i\tau_2 \Phi_i^\star \\ &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_i + \eta_i - i\chi_i) \\ -\varphi_i^- \end{pmatrix}, \quad i = 1, 2 \end{aligned} \tag{3.14}$$

There are different types of the 2HDM (type-I, type-II,...) due to different ways of coupling the Higgs fields to up-type, and down-type quarks and leptons.

The famous parameter $\tan\beta$ in generic basis has no physical meaning, because no physical Higgs coupling depends on $\tan\beta$. The general 2HDM generally predicts FCNCs in conflict with the experimental data. One way to avoid this phenomenological problem is to constrain the theoretical structure of the 2HDM. The most common constraint to impose on the 2HDM is a requirement that some of the Higgs-fermion Yukawa couplings vanish in a "preferred" basis. This leads to the well known type-I and type-II models. In the 2HDM-I the Higgs-fermion couplings in the "preferred" basis, $g_2^u = g_2^d = 0$ and in the 2HDM-II "preferred" basis, $g_1^u = g_2^d = 0$ [69, 70]. The parameter $\tan\beta$ in these cases is basis-independent, so $\tan\beta$ is indeed physical [71].

3.2.1 Yukawa Lagrangian of 2HDM type-II

This is an important version of the 2HDM, because FCNC is forbidden in the "preferred" basis, $g_1^u = g_2^d = 0$ [69, 70]. The up-type quarks get their masses from the expectation value of the second doublet Φ_2 while the down-type quarks get their masses from the first doublet Φ_1 like in the SM, also it is similar to the MSSM [13].

The Yukawa Lagrangian of 2HDM type-II is written as [67]

$$\begin{aligned} -\mathcal{L}_{II}^Y &= g_1^d \big[\bar{\Psi}_q^L \psi_d^R \Phi_1 + \bar{\psi}_d^R \Phi_1^\dagger \Psi_q^L\big] \\ &+ g_2^u \big[\bar{\Psi}_q^L \psi_u^R \tilde{\Phi}_2 + \bar{\psi}_u^R \tilde{\Phi}_2^\dagger \Psi_q^L\big] \\ &+ g_1^e \big[\bar{\Psi}_l^L \psi_l^R \Phi_1 + \bar{\psi}_l^R \Phi_1^\dagger \Psi_l^L\big] \end{aligned} \tag{3.15}$$

or

$$
\begin{aligned}
-\mathcal{L}_{II}^{Y} &= g_1^d\big[\bar\psi_u^L\psi_d^R\varphi_1^+ + \bar\psi_d^R\psi_u^L\varphi_1^- \\
&\quad + \frac{\bar\psi_d^L\psi_d^R}{\sqrt{2}}(v_1+\eta_1+i\chi_1) + \frac{\bar\psi_d^R\psi_d^L}{\sqrt{2}}(v_1+\eta_1-i\chi_1)\big] \\
&\quad + g_2^u\big[-\bar\psi_d^L\psi_u^R\varphi_2^- - \bar\psi_u^R\psi_d^L\varphi_2^+ \\
&\quad + \frac{\bar\psi_u^L\psi_u^R}{\sqrt{2}}(v_2+\eta_2-i\chi_2) + \frac{\bar\psi_u^R\psi_u^L}{\sqrt{2}}(v_2+\eta_2+i\chi_2)\big] \\
,&\quad + g_1^e\big[\bar\psi_{\nu_l}^L\psi_l^R\varphi_1^+ + \bar\psi_l^R\psi_{\nu_l}^L\varphi_1^- \\
&\quad + \frac{\bar\psi_l^L\psi_l^R}{\sqrt{2}}(v_1+\eta_1+i\chi_1) + \frac{\bar\psi_l^R\psi_l^L}{\sqrt{2}}(v_1+\eta_1-i\chi_1)\big] \qquad (3.16)
\end{aligned}
$$

The non-zero v.e.v.'s v_1 and v_2 of the two doublets give masses to the fermions such as

$$
\begin{aligned}
-\mathcal{L}_{II(\mathrm{mass})}^{Y} &= \frac{g_1^d}{\sqrt{2}}(\bar\psi_d^L\psi_d^R + \bar\psi_d^R\psi_d^L)v_1 + \frac{g_2^u}{\sqrt{2}}(\bar\psi_u^L\psi_u^R + \bar\psi_u^R\psi_u^L)v_2 \\
&\quad + \frac{g_1^e}{\sqrt{2}}(\bar\psi_l^L\psi_l^R + \bar\psi_l^R\psi_l^L)v_1 \\
&= \frac{g_1^d}{\sqrt{2}}(\bar\psi_d\psi_d)v_1 + \frac{g_2^u}{\sqrt{2}}(\bar\psi_u\psi_u)v_2 + \frac{g_1^e}{\sqrt{2}}(\bar\psi_l\psi_l)v_1 \qquad (3.17)
\end{aligned}
$$

The masses of the fermions can be explicitly identified as

$$
m_u = \frac{g_2^u}{\sqrt{2}}v\sin\beta, \quad m_d = \frac{g_1^d}{\sqrt{2}}v\cos\beta, \quad m_e = \frac{g_1^e}{\sqrt{2}}v\cos\beta \qquad (3.18)
$$

In the following sections we will discuss the CP-conserving case in the 2HDM.

3.3 CP conservation in the 2HDM

In the minimal SM the Higgs sector comprises only one complex Higgs doublet [1] resulting in one physical neutral Higgs scalar whose mass is a free parameter of the theory. There is no experimental evidence for the SM Higgs, the theory fails to explain the observed size of the baryon asymmetry in the universe. Therefore it is important to study the extended models containing more than one physical Higgs boson in the spectrum.

One of the earliest reasons for introducing the 2HDM was to describe the phenomenon of CP violation [8]. The additional Higgs doublet gives a possibility of FCNC which is highly suppressed relative to the charged current processes, so it would be desirable to suppress it from the 2HDM. If all quarks with the same quantum numbers couple to the same scalar doublet, then FCNC will be absent. This led Glashow and Weinberg [62] to propose a discrete symmetry or Z_2 symmetry, which force all the quarks of a given charge to couple to only one doublet. The Lagrangian is invariant under the interchange

$$
\begin{aligned}
\Phi_1 \leftrightarrow \Phi_1 \quad &, \quad \Phi_2 \leftrightarrow -\Phi_2, \quad \text{or} \\
\Phi_1 \leftrightarrow -\Phi_1 \quad &, \quad \Phi_2 \leftrightarrow \Phi_2. \qquad (3.19)
\end{aligned}
$$

This symmetry forbids the $\Phi_1 \leftrightarrow \Phi_2$ transition.

There are three cases of the Z_2 symmetry as follows

- Exact Z_2 symmetry. This is the case of CP-conservation in the 2HDM, where $\lambda_6 = \lambda_7 = m_{12} = 0$ or real and λ_5 is real, this requires in the $\Phi_1 - \Phi_2$ basis, all the parameters of the Higgs potential are real. The neutral Higgs bosons mass matrix is diagonalized and the Higgs field rotated to the physical Higgs bosons, two neutral CP-even (CP=1, mixtures of the real parts of the neutral Higgs fields) h, and H with $m_h \leq m_H$ and one CP-odd neutral Higgs A (CP=-1, derives from the imaginary components which are not eaten by the Z), also the charged Higgs mass matrix is diagonalized to give two charged Higgs particles $H^{\pm}$. The parameters of the Higgs potential in this case are 6 parameters.

- Soft Z_2 symmetry violation. A symmetry is said to be softly broken when all terms which break it have dimension two. This case corresponds to the explicit CP-violation where both m_{12} and λ_5 are complex. This type of Z_2 violation respects the Z_2 symmetry at small distances in all orders of perturbation theory.

- Hard Z_2 symmetry violation. A symmetry is said to be hard broken when all terms which break it have dimension two and four. This is the general case with more CP violation, where λ_6, and λ_7 are complex in addition to the soft Z_2 symmetry violation. The total parameters of the Higgs potential in this case are 14.

CP-conservation in the Higgs sector means that there is no explicit or spontaneous breaking in the 2HDM (exact Z_2 symmetry). In the decoupling limit, where ($m_H^{\pm} \gg v$) the lightest neutral Higgs boson h has mass of $\mathcal{O}(v)$, while the other two neutral Higgs bosons have mass of $\mathcal{O}(m_H^{\pm})$. One can formally integrate out the heavy Higgs states from the theory [72, 73]. The resulting Higgs effective theory yields precisely the SM Higgs sector up to corrections of $\mathcal{O}(v^2/m_H^{\pm})$. Thus the properties of the light Higgs h are nearly identical to those of the CP-even SM Higgs boson.

The most general CP-invariant Higgs potential having two complex $Y = 1$, $SU(2)_L$ doublet scalar fields Φ_1 and Φ_2 is given by [13]

$$\begin{aligned} V &= \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \Big[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\Big]\,\mathrm{Re}\,(\Phi_1^\dagger\Phi_2) \\ &-\frac{1}{2}\Big\{m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{12}^2\mathrm{Re}\,(\Phi_1^\dagger\Phi_2) + m_{22}^2(\Phi_2^\dagger\Phi_2)\Big\}\,. \end{aligned} \tag{3.20}$$

where $\lambda_5, \lambda_6, \lambda_7$ and m_{12} are real.

It is convenient to expand the Higgs-doublet fields about their vacuum states as

$$\Phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix} \tag{3.21}$$

and choose phases of Φ_i such that v_1 and v_2 are both real [38], where φ_i^+ are complex fields, and η_i and χ_i are real fields. This, in turn enables us to write the mass terms of the potential (3.20) as:

$$V_{\mathrm{mass}} = \begin{pmatrix}\varphi_1^- & \varphi_2^-\end{pmatrix}\mathcal{M}^2_{\mathrm{ch}}\begin{pmatrix}\varphi_1^+ \\ \varphi_2^+\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\chi_1 & \chi_2\end{pmatrix}\mathcal{M}^2_{\text{CP-odd}}\begin{pmatrix}\chi_1 \\ \chi_2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\eta_1 & \eta_2\end{pmatrix}\mathcal{M}^2_{\text{CP-even}}\begin{pmatrix}\eta_1 \\ \eta_2\end{pmatrix} \tag{3.22}$$

where $\mathcal{M}^2_{\text{ch}}$ is the squared mass matrix of the charged sector, $\mathcal{M}^2_{\text{CP-odd}}$ is the squared mass matrix of the CP-odd sector and $\mathcal{M}^2_{\text{CP-even}}$ is the squared mass matrix of the CP-even Higgs sector.

Diagonalizing the quadratic terms of the V_{mass}, one obtains the mass eigenstates: 2 neutral CP-even scalar particles, h and H, a neutral CP-odd A, charged fields $H^\pm$. The relations between the mass eigenstates and the $SU(2) \times U(1)$ eigenstates are:

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = R_\beta \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ A \end{pmatrix} = R_\beta \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \quad \begin{pmatrix} h \\ H \end{pmatrix} = R_\alpha \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \tag{3.23}$$

or

$$\begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix} = R_\beta^{-1} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}, \quad \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = R_\beta^{-1} \begin{pmatrix} G^0 \\ A \end{pmatrix}, \quad \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R_\alpha^{-1} \begin{pmatrix} h \\ H \end{pmatrix} \tag{3.24}$$

with

$$R_\beta = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}, \quad R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{3.25}$$

where $\sin\beta = s_\beta$ and $\cos\beta = c_\beta$.

One can derive the squared mass matrix of the charged Higgs sector (using Reduce or Mathematica program). First extract the charged part of the Higgs potential by setting the neutral Higgs fields to be zero ($\eta_1 = \eta_2 = \chi_1 = \chi_2 = 0$). The squared mass matrix $\mathcal{M}^2_{ch}$ (2×2) elements of the charged Higgs sector are then extracted from the coefficients of two charged Higgs fields ($\varphi^+\varphi^-$), e.g., the matrix element M_{ch11} is the coeficient of the fields $\varphi_1^+\varphi_1^-$. Thus the squared mass matrix of the charged Higgs sector is found to be

$$\begin{aligned}
\mathcal{M}^2_{11\text{ch}} &= -\frac{s_\beta}{2c_\beta} v^2 \Big(s_\beta c_\beta(\lambda_4 + \lambda_5) + c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7 \Big) + \frac{s_\beta}{2c_\beta} m_{12}^2, \\
\mathcal{M}^2_{22\text{ch}} &= -\frac{c_\beta}{2s_\beta} v^2 \Big(s_\beta c_\beta(\lambda_4 + \lambda_5) + c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7 \Big) + \frac{c_\beta}{2s_\beta} m_{12}^2, \\
\mathcal{M}^2_{12\text{ch}} &= -\frac{v^2}{2} \Big(s_\beta c_\beta(\lambda_4 + \lambda_5) + c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7 \Big) - \frac{1}{2} m_{12}^2
\end{aligned} \tag{3.26}$$

with $\mathcal{M}^2_{ji} = \mathcal{M}^2_{ij}$.

The angle β is used to diagonalize the charged Higgs fields as

$$R_\beta \mathcal{M}^2_{ch} R_\beta^T = \mathcal{M}^2_{\text{ch(diag.)}} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \mathcal{M}^2_{11\text{ch}} & \mathcal{M}^2_{12\text{ch}} \\ \mathcal{M}^2_{21\text{ch}} & \mathcal{M}^2_{22\text{ch}} \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \tag{3.27}$$

The charged Higgs mass can be derived as

$$m^2_{H^\pm} = \mu^2 - \frac{v^2}{2}(\lambda_4 + \lambda_{567}) \tag{3.28}$$

Here and in the following we will use the abbreviations

$$\begin{gathered}
\lambda_{567} = \lambda_5 + \frac{v_1}{v_2}\lambda_6 + \frac{v_2}{v_1}\lambda_7, \quad m_{12}^2 = 2\mu^2 c_\beta s_\beta \\
\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \quad \lambda_{34567} = \lambda_{345} + \frac{v_1}{v_2}\lambda_6 + \frac{v_2}{v_1}\lambda_7, \quad \mu^2 = v^2\nu.
\end{gathered} \tag{3.29}$$

The angle α is used to diagonalize the CP-even Higgs fields as follows,

$$
\begin{aligned}
\begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix} &= R_\alpha \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 \end{pmatrix} R_\alpha^T \\
&= \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 \\ \mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \qquad (3.30)
\end{aligned}
$$

The squared mass matrix $\mathcal{M}^2$ elements of (3.30), corresponding to the neutral sector of the potential, is found to be

$$
\begin{aligned}
\mathcal{M}_{11}^2 &= v^2[c_\beta^2\,\lambda_1 + s_\beta^2\,\nu + \frac{s_\beta}{2c_\beta}(3c_\beta^2\,\lambda_6 - s_\beta^2\,\lambda_7)], \\
\mathcal{M}_{22}^2 &= v^2[s_\beta^2\,\lambda_2 + c_\beta^2\,\nu + \frac{c_\beta}{2s_\beta}(-c_\beta^2\,\lambda_6 + 3s_\beta^2\,\lambda_7)], \\
\mathcal{M}_{33}^2 &= v^2[-\lambda_5 + \nu - \frac{1}{2c_\beta s_\beta}(c_\beta^2\,\lambda_6 + s_\beta^2\,\lambda_7)], \\
\mathcal{M}_{12}^2 &= v^2[c_\beta s_\beta(\lambda_{345} - \nu) + \tfrac{3}{2}(c_\beta^2\,\lambda_6 + s_\beta^2\,\lambda_7)],
\end{aligned}
\qquad (3.31)
$$

with $\mathcal{M}_{ji}^2 = \mathcal{M}_{ij}^2$.

The spectral masses of the neutral Higgs bosons are obtained by diagonalizing the mass matrix. One finds, diagonalizing the respective 2×2 matrices, the CP-odd Higgs boson mass is

$$
M_A^2 = \mu^2 - v^2\Big[\lambda_5 + \frac{1}{2c_\beta s_\beta}(c_\beta^2\lambda_6 + s_\beta^2\lambda_7)\Big] = \mu^2 - \frac{1}{2}v^2[\lambda_5 + \lambda_{567}] \qquad (3.32)
$$

The masses of the CP-even neutral Higgs are

$$
M_{h,H}^2 = \frac{1}{2}\Big(M_{11} + M_{22} \mp \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}\Big) \qquad (3.33)
$$

with

$$
M_H^2 + M_h^2 = v^2\Big(c_\beta^2\lambda_1 + s_\beta^2\lambda_2 + \nu + \frac{1}{2}[(3t_\beta - t_\beta^{-1})c_\beta^2\lambda_6 + (3t_\beta^{-1} - t_\beta)s_\beta^2\lambda_7]\Big) \qquad (3.34)
$$

and

$$
M_H^2 - M_h^2 = \frac{v^2[\sin 2\beta(\lambda_{345} - \nu) + 3(c_\beta^2\lambda_6 + s_\beta^2\lambda_7)]}{\sin 2\alpha} \qquad (3.35)
$$

where

$$
\sin 2\alpha = \frac{2M_{12}}{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}}, \quad \cos 2\alpha = \frac{M_{11} - M_{22}}{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2}}, \quad t_\beta = \tan\beta \qquad (3.36)
$$

To avoid the FCNC, one takes $\lambda_6 = \lambda_7 = 0$. The squared mass matrix (3.31) can then be rewritten as

$$
\mathcal{M}^2 = v^2 \begin{pmatrix} \lambda_1 c_\beta^2 + \nu\, s_\beta^2 & (\lambda_{345} - \nu)c_\beta\, s_\beta & 0 \\ (\lambda_{345} - \nu)c_\beta\, s_\beta & \lambda_2\, s_\beta^2 + \nu\, c_\beta^2 & 0 \\ 0 & 0 & -\lambda_5 + \nu \end{pmatrix} \qquad (3.37)
$$

And the squared mass matrix (3.26) is reduced to

$$\begin{aligned}
\mathcal{M}^2_{11ch} &= -\frac{s_\beta}{2c_\beta}v^2\Big(s_\beta c_\beta(\lambda_4+\lambda_5)\Big)+\frac{s_\beta}{2c_\beta}m^2_{12},\\
\mathcal{M}^2_{22ch} &= -\frac{c_\beta}{2s_\beta}v^2\Big(s_\beta c_\beta(\lambda_4+\lambda_5)\Big)+\frac{c_\beta}{2s_\beta}m^2_{12},\\
\mathcal{M}^2_{12ch} &= -\frac{v^2}{2}\Big(s_\beta c_\beta(\lambda_4+\lambda_5)\Big)-\frac{1}{2}m^2_{12}
\end{aligned} \tag{3.38}$$

with $\mathcal{M}^2_{ji}=\mathcal{M}^2_{ij}$.

In the next section we will study the soft CP-violation.

3.4 CP-violation in the 2HDM

CP violation plays an important role in our understanding of cosmology. This is because of the observed baryon asymmetry of the universe. Another interesting consequence of CP violation would be the possibility that the elementary particles have electric dipole moments. CP-violation in the SM via the complex phase in the CKM matrix is very small and not enough to explain the baryon asymmetry in the universe, therefore some extension of the SM is important. The 2HDM is one of the promising models because it provides a rich set of possibilities for CP violation in addition to that from the CKM matrix in the SM. The sources of CP violation in 2HDM can be summarized as [6],

- Usual CKM matrix complex phase
- CP violation in the charged-Higgs exchange, e.g., charged Higgs exchange in $B-\bar{B}$ mixing and $D-\bar{D}$ mixing.
- CP violation in the additional neutral-Higgs exchange e.g., the parameters of the 2HDM potential are complex.

Neutron electric dipole moment (NEDM) is a consequence of CP violation through the exchange of the neutral Higgs bosons [14]. Also Weinberg proposed a gauge theory of CP nonconservation through the exchange of the charged Higgs bosons [15].

Higgs sector CP violation may be either explicit or spontaneous.

- Explicit CP violation. It means that the Lagrangian or the Higgs potential is not invariant under the CP transformation. This type of CP violation occurs when the Higgs potential breaks the Z_2 symmetry softly i.e., λ_5 is complex and $\mathrm{Im}\,(m_{12}) \neq 0$ or it breaks the Z_2 symmetry hardly i.e., λ_6 and λ_7 also are complex. This CP violation is responsible for mixing different CP states (CP-even and CP-odd). There is another source of CP violation from the Yukawa couplings for the neutral and the charged sectors that will be discussed later.
- Spontaneous CP violation. If the scalar Lagrangian is explicitly CP conserving, but the vacuum state of the theory violates CP, then one can say that CP is spontaneously broken. After spontaneous symmetry breaking in the 2HDM, it is natural

to have spontaneous CP violation. Early papers claimed that a relative phase between the v.e.v.'s of the two doublets would be responsible for spontaneous CP violation [74]. But according to Branco and his collaborators the phases of the Higgs field can often be chosen such that the v.e.v.'s are real and positive. Such phase difference between the two v.e.v.'s then has no physical meaning and can be eliminated by rephasing the fields [38, 75]. In the SM there is only one doublet, so there is no spontaneous CP violation.

The CP-violating couplings of the lightest neutral Higgs boson to the fermions, gauge bosons and to itself are suppressed by a factor of $\mathcal{O}(v^2/m_{H^\pm})$ if we consider the charged Higgs is to be very heavy (see [19]).

The 2HDM may be seen as an unconstrained version of the Higgs sector of the MSSM. While at tree level the latter can be parametrized in terms of only two parameters, conventionally taken to be $\tan\beta$ and M_A, the 2HDM has much more freedom. In particular, the neutral and charged Higgs masses are rather independent. Traditionally, the 2HDM is defined in terms of the potential. The parameters of the potential (quartic and quadratic couplings) determine the masses of the neutral and the charged Higgs bosons [76].

In addition, the 2HDM neutral sector may or may not lead to CP violation, depending on the choice of potential. In the most general CP-violating 2HDM, the physical Higgs fields are H_1, H_2 and H_3. In this section one considers soft Z_2 symmetry violation, where $\mathrm{Im}\, m_{12}^2 \neq 0$ and λ_5 are complex, therefore FCNC at tree-level is very small and suppressed. The most general CP nonconserving Higgs potential having two complex $SU(2)_L$ doublet scalar fields Φ_1 and Φ_2 is given by [13] with $Y = 1$. We shall here consider the so-called Model II, where u-type quarks acquire masses from a Yukawa coupling to one Higgs doublet Φ_2, whereas the d-type quarks couple to the other Φ_1. This structure is the same as in the MSSM.

We will try to study the CP violation in two different cases according to the Z_2 symmetry breaking. In this section we will start by the Z_2 softly symmetry breaking case ($\mathrm{Im}\,\lambda_6 = \mathrm{Im}\,\lambda_7 = 0$), and in the next section we will study the general case where Z_2 symmetry is hardly broken ($\mathrm{Im}\,\lambda_6 \neq 0$, $\mathrm{Im}\,\lambda_7 \neq 0$). In the 2HDM with CP violation, the physical mass eigenstates, H_i ($i = 1, 2, 3$), are mixtures (specified by three mixing angles α_i, $i = 1, 2, 3$) of the real and imaginary components of the original neutral Higgs doublet fields; as a result H_i have undefined CP properties. The special case of the potential (3.20) can take the form

$$\begin{aligned} V \;=\; & \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] \\ & - \frac{1}{2}\left\{m_{11}^2(\Phi_1^\dagger\Phi_1) + \left[m_{12}^2(\Phi_1^\dagger\Phi_2) + \text{h.c.}\right] + m_{22}^2(\Phi_2^\dagger\Phi_2)\right\}. \end{aligned} \tag{3.39}$$

where λ_5 and m_{12}^2 are complex. It is convenient to define $\eta_3 = -\sin\beta\chi_1 + \cos\beta\chi_2$ orthogonal to the neutral Goldstone boson $G^0 = \cos\beta\chi_1 + \sin\beta\chi_2$. In the basis ($\eta_1$, η_2, η_3), the resulting squared mass matrix $\mathcal{M}^2$ of the neutral sector, can then be diagonalized to physical states $(H_1,\ H_2,\ H_3)$ with masses $M_1 \leq M_2 \leq M_3$, via a rotation matrix R:

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}, \tag{3.40}$$

satisfying

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2), \tag{3.41}$$

and parametrized as

$$\begin{aligned} R = R_3\, R_2\, R_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_3 & \sin\alpha_3 \\ 0 & -\sin\alpha_3 & \cos\alpha_3 \end{pmatrix} \begin{pmatrix} \cos\alpha_2 & 0 & \sin\alpha_2 \\ 0 & 1 & 0 \\ -\sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 \\ -\sin\alpha_1 & \cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix} \end{aligned} \tag{3.42}$$

with $c_i = \cos\alpha_i$, $s_i = \sin\alpha_i$. The rotation angle α_1 is chosen such that in a particular limit of no CP violation $s_2 \to 0$, $s_3 \to 0$, then $\alpha_1 \to \alpha + \frac{1}{2}\pi$, where $\alpha(-\pi/2 < \alpha \leq 0)$ is the familiar mixing angle of the CP-even sector [77]. The additional $\frac{1}{2}\pi$ provides the mapping $H_1 \leftrightarrow h$, instead of H being in the $(1,1)$ position of $\mathcal{M}^2_{\mathrm{diag}}$, as used in the MSSM [13]. The phase of H_i has no physical consequence, thus one may freely change the sign of one or more rows, e.g., let $R_{1i} \to -R_{1i}$, see Ref. [17].

For the case of the potential (3.39), the squared mass matrix (3.37) is generalized to the CP violation case and can be written as

$$M^2 = v^2 \begin{pmatrix} \lambda_1 c_\beta^2 + \nu\, s_\beta^2 & (\mathrm{Re}\,\lambda_{345} - \nu) c_\beta\, s_\beta & -\frac{1}{2}\,\mathrm{Im}\,\lambda_5\, s_\beta \\ (\mathrm{Re}\,\lambda_{345} - \nu) c_\beta\, s_\beta & \lambda_2\, s_\beta^2 + \nu\, c_\beta^2 & -\frac{1}{2}\,\mathrm{Im}\,\lambda_5\, c_\beta \\ -\frac{1}{2}\,\mathrm{Im}\,\lambda_5\, s_\beta & -\frac{1}{2}\,\mathrm{Im}\,\lambda_5\, c_\beta & -\mathrm{Re}\,\lambda_5 + \nu \end{pmatrix} \tag{3.43}$$

Rather than describing the phenomenology in terms of the parameters of the potential Eq. (3.39), in [78] the physical mass of the charged Higgs boson, as well as those of the two lightest neutral ones, were taken as input, together with the rotation matrix R. Thus, the input can be summarized as

$$\text{Parameters: } \tan\beta,\ (M_1,\ M_2),\ (M_{H^\pm},\ \mu^2),\ (\alpha_1,\ \alpha_2,\ \alpha_3). \tag{3.44}$$

This approach is used also here and provides better control of the physical content of the model. In particular, the elements R_{13} and R_{23} of the rotation matrix must be non-zero in order to yield CP violation. For consistency, this requires $\mathrm{Im}\,\lambda_5$ and $\mathrm{Im}\,m_{12}^2$ (as derived quantities) to be non-zero. This approach highlights the fact that the neutral and charged sectors are rather independent, as well as masses being physically more accessible than quartic couplings. However, some choices of input will lead to physically acceptable potentials, others will not. In this way, the two sectors remain correlated [17].

In the next section we will study the general case of the 2HDM type-II where λ_6 and λ_7 are non-zero.

3.5 The general potential

The potential for CP violation can be obtained if one takes the parameters of (3.20) to be complex. The terms proportional to λ_6 and λ_7 have to be carefully constrained, since this potential does not satisfy natural flavour conservation [62], even if each doublet is coupled only to up-type or only to down-type flavours.

The various coupling constants in the potential will of course depend on the choice of basis (Φ_1, Φ_2). Recently, there has been some focus [79] on the importance of formulating physical observables in a basis-independent manner. Here, we shall adopt the so-called Model II [13] for the Yukawa couplings. This will uniquely identify the basis in the (Φ_1, Φ_2) space.

3.5.1 Reparameterizing the 2HDM potential

The 2HDM potential is invariant under the global transformation of the fields [38]

$$\Phi_i \to e^{-i\rho}\Phi_i, \quad \rho_i \quad \text{real} \quad (i = 1, 2) \tag{3.45}$$

accompanied by the following redefinition of parameters:

$$\begin{aligned} \lambda_{1-4} &\to \lambda_{1-4}, \quad m_{11(22)}^2 \to m_{11(22)}^2, \\ \lambda_5 &\to \lambda_5 e^{2i(\rho_2-\rho_1)}, \quad \lambda_{6,7} \to \lambda_{6,7} e^{i(\rho_2-\rho_1)}, \quad \text{and} \quad m_{12}^2 \to m_{12}^2 e^{i(\rho_2-\rho_1)}. \end{aligned} \tag{3.46}$$

In order to have $U(1)$ symmetry of electromagnetism, the v.e.v.'s can be chosen as [38, 80]

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \text{and} \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} e^{i\xi} \end{pmatrix} \tag{3.47}$$

The phase difference ξ is known as spontaneous CP violation parameter. Under rephasing, Eq.(3.45), the phase ξ is changed according to

$$\xi \to \xi + \rho_1 - \rho_2. \tag{3.48}$$

Therefore the quantities

$$\lambda_5 \to \lambda_5 e^{2i\xi}, \quad \lambda_{6,7} \to \lambda_{6,7} e^{i\xi}, \quad \text{and} \quad m_{12}^2 \to m_{12}^2 e^{i\xi}, \tag{3.49}$$

are rephasing-invariant quantities [81].

Minimization of the 2HDM potential

The minimum of the potential defines the v.e.v.'s of the fields Φ_i as

$$\left.\frac{\partial V}{\partial \Phi_1}\right|_{\substack{\Phi_1=\langle\Phi_1\rangle,\\ \Phi_2=\langle\Phi_2\rangle}} = 0, \quad \left.\frac{\partial V}{\partial \Phi_2}\right|_{\substack{\Phi_1=\langle\Phi_1\rangle,\\ \Phi_2=\langle\Phi_2\rangle}} = 0 \tag{3.50}$$

And by using the following relations

$$\mathrm{Im}(z^\dagger e^{-i\xi}) = -\mathrm{Im}(z e^{i\xi}), \quad \text{and} \quad \mathrm{Re}(z^\dagger e^{-i\xi}) = \mathrm{Re}(z e^{i\xi}) \tag{3.51}$$

where z is any complex number, one can eliminate m_{11}^2, m_{22}^2 and m_{12}^2 as follows

$$\begin{aligned}
m_{11}^2 = &\quad \lambda_1 v_1^2 + [\lambda_3 + \lambda_4 + \mathrm{Re}(\lambda_5 e^{2i\xi})]v_2^2 + \frac{v_2}{v_1}\mathrm{Re}[3(\lambda_6 e^{i\xi})v_1^2 + (\lambda_7 e^{i\xi})v_2^2 - (m_{12}^2 e^{i\xi})] \\
m_{22}^2 = &\quad \lambda_2 v_2^2 + [\lambda_3 + \lambda_4 + \mathrm{Re}(\lambda_5 e^{2i\xi})]v_1^2 + \frac{v_1}{v_2}\mathrm{Re}[(\lambda_6 e^{i\xi})v_1^2 + 3(\lambda_7 e^{i\xi})v_2^2 - (m_{12}^2 e^{i\xi})]
\end{aligned} \tag{3.52}$$

and

$$\mathrm{Im}(m_{12}^2 e^{i\xi}) = \mathrm{Im}[(\lambda_5 e^{2i\xi})v_1 v_2 + (\lambda_6 e^{i\xi})v_1^2 + (\lambda_7 e^{i\xi})v_2^2]. \tag{3.53}$$

Without loss of generality, one can put $\xi = 0$. Therefore the potential (3.20) with complex parameter, can be rewritten (modulo a constant) as

$$\begin{aligned}
V \;=\; & \frac{\lambda_1}{2}\left[(\Phi_1^\dagger\Phi_1) - \frac{v_1^2}{2}\right]^2 + \frac{\lambda_2}{2}\left[(\Phi_2^\dagger\Phi_2) - \frac{v_2^2}{2}\right]^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \mathrm{h.c.}\right\} \\
& -\frac{1}{2}\left[\mathrm{Re}\,\lambda_{34567} - 2\nu\right]\left[v_2^2(\Phi_1^\dagger\Phi_1) + v_1^2(\Phi_2^\dagger\Phi_2)\right] - v_1 v_2\,\mathrm{Re}\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right] \\
& - v_1 v_2\left[2\nu\,\mathrm{Re}\,(\Phi_1^\dagger\Phi_2) - \mathrm{Im}\,\lambda_{567}\,\mathrm{Im}\,(\Phi_1^\dagger\Phi_2)\right].
\end{aligned} \tag{3.54}$$

The potential contains terms which violate charge conjugation (C) and are odd under complex conjugation of the fields Φ_1 and Φ_2 [82], i.e., mixing terms. When the fields Φ_1 and Φ_2 are coupled to fermions, these C-violating terms lead to CP violation (see [38]). There are two types of mixing terms, quartic (originating from non-zero Imλ_5, Imλ_6, and Imλ_7) and quadratic (originating from $\mathrm{Im}(m_{12}^2)$). From (3.53), one cannot have C (or CP) violation by quadratic terms only, they will always be accompanied by quartic terms.

The full diagonalization is achieved with

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \tag{3.55}$$

Thus, the physical masses of the neutral Higgs bosons are given by

$$\begin{pmatrix} M_1^2 & & \\ & M_2^2 & \\ & & M_3^2 \end{pmatrix} = R \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{21}^2 & M_{22}^2 & M_{23}^2 \\ M_{31}^2 & M_{32}^2 & M_{33}^2 \end{pmatrix} R^T \tag{3.56}$$

The squared mass matrix $\mathcal{M}^2$ of (3.41), corresponding to the neutral sector of the potential, can be derived by differentiating the potential with respect to the weak basis fields and setting these fields equal to zero i.e., $\mathcal{M}^2_{ij} = \frac{\partial^2 V}{\partial\eta_i\partial\eta_j}\,|_{\eta_1=\eta_2=\eta_3=0}$. One finds

$$\begin{aligned}
\mathcal{M}^2_{11} &= v^2[c^2_\beta\,\lambda_1 + s^2_\beta\,\nu + \frac{s_\beta}{2c_\beta}\mathrm{Re}\,(3c^2_\beta\,\lambda_6 - s^2_\beta\,\lambda_7)],\\
\mathcal{M}^2_{22} &= v^2[s^2_\beta\,\lambda_2 + c^2_\beta\,\nu + \frac{c_\beta}{2s_\beta}\mathrm{Re}\,(-c^2_\beta\,\lambda_6 + 3s^2_\beta\,\lambda_7)],\\
\mathcal{M}^2_{33} &= v^2\mathrm{Re}\,[-\lambda_5 + \nu - \frac{1}{2c_\beta s_\beta}(c^2_\beta\,\lambda_6 + s^2_\beta\,\lambda_7)],\\
\mathcal{M}^2_{12} &= v^2[c_\beta s_\beta(\mathrm{Re}\,\lambda_{345} - \nu) + \tfrac{3}{2}\mathrm{Re}\,(c^2_\beta\,\lambda_6 + s^2_\beta\,\lambda_7)],\\
\mathcal{M}^2_{13} &= -\frac{1}{2}v^2\mathrm{Im}\,[s_\beta\,\lambda_5 + 2c_\beta\,\lambda_6],\\
\mathcal{M}^2_{23} &= -\frac{1}{2}v^2\mathrm{Im}\,[c_\beta\,\lambda_5 + 2s_\beta\,\lambda_7],
\end{aligned}\tag{3.57}$$

with $\mathcal{M}^2_{ji} = \mathcal{M}^2_{ij}$.

Here, compared with the potential (3.39), we have two more complex parameters, λ_6 and λ_7 (four new real parameters), but rather than those, we take as additional parameters M_3, $\mathrm{Im}\,\lambda_5$, $\mathrm{Re}\,\lambda_6$ and $\mathrm{Re}\,\lambda_7$. Thus, the input will be

$$\text{Parameters: } \tan\beta,\ (M_1,\ M_2,\ M_3),\ (M_{H^\pm},\ \mu^2),\ (\alpha_1,\ \alpha_2,\ \alpha_3),\ \mathrm{Im}\,\lambda_5,\ (\mathrm{Re}\,\lambda_6,\ \mathrm{Re}\,\lambda_7). \tag{3.58}$$

In the next section we try to study some special cases of the Yukawa coupling.

3.6 $H_j t\bar{t}$ and $H_j b\bar{b}$ coupling

The Yukawa interactions couple the Higgs fields to a left-handed doublet and a right-handed singlet quark field. However, these do not need to be in the flavor basis in which the mass matrices are diagonal. The Z_2 symmetry, which is imposed to stabilize Model II [15, 62] is broken by the m^2_{12} and $\mathrm{Im}\,\lambda_5$ terms, as well as by the λ_6 and λ_7 terms, therefore one should be careful when dealing with these terms because they can produce FCNC. The discovery of the top quark and measuring its mass $m_t \approx v/\sqrt{2}$ plays a significant role in probing the new physics beyond the SM. The process $e^+e^- \to t\bar{t}H_j$ is an ideal process for probing anomalous coupling $H_j t\bar{t}$. Also the $H_j t\bar{t}$ coupling can be studied in the process $gg \to t\bar{t}$ by exchanging of the non-standard neutral Higgs boson.

By transforming the Higgs fields into the physical basis, see (3.55), and using (3.16), the Lagrangian of the t-quark Yukawa coupling can be rewritten as

$$\begin{aligned}
-\mathcal{L}^Y_{II} &= \frac{m_t}{v\sin\beta}\big(\bar{\psi}^L_t(\eta_2 - i\chi_2)\psi^R_t + \bar{\psi}^R_t(\eta_2 + i\chi_2)\psi^L_t\big) + \cdots\\
&= \frac{m_t}{2v\sin\beta}\big[\bar{\psi}_t(1+\gamma^5)\psi_t(R_{j2}H_j - i\cos\beta R_{j3}H_j)\\
&\quad +\bar{\psi}_t(1-\gamma^5)\psi_t(R_{j2}H_j + i\cos\beta R_{j3}H_j)\big] + \cdots\\
&= \frac{m_t}{v\sin\beta}\big[\bar{\psi}_t\psi_t R_{j2}H_j - i\gamma^5\cos\beta\bar{\psi}_t\psi_t R_{j3}H_j\big] + \cdots
\end{aligned}\tag{3.59}$$

For the $H_j b\bar{b}$ coupling

$$
\begin{aligned}
-\mathcal{L}_{II}^{Y} &= \frac{m_b}{v\cos\beta}\big[\bar{\psi}_b^L(\eta_1+i\chi_1)\psi_b^R+\bar{\psi}_b^R(\eta_1-i\chi_1)\psi_b^L\big]+\cdots \\
&= \frac{m_b}{2v\cos\beta}\big[\bar{\psi}_b(1+\gamma^5)\psi_b(R_{j1}H_j-i\sin\beta R_{j3}H_j) \\
&\quad +\bar{\psi}_b(1-\gamma^5)\psi_b(R_{j1}H_j+i\sin\beta R_{j3}H_j)\big]+\cdots \\
&= \frac{m_b}{v\cos\beta}\big[\bar{\psi}_b\psi_b R_{j1}H_j-i\gamma^5\sin\beta\bar{\psi}_b\psi_b R_{j3}H_j\big]+\cdots \qquad (3.60)
\end{aligned}
$$

The couplings can be expressed (relative to the SM coupling) as

$$
\begin{aligned}
H_j b\bar{b}: &\qquad \frac{1}{\cos\beta}\,[R_{j1}-i\gamma_5\sin\beta R_{j3}], \\
H_j t\bar{t}: &\qquad \frac{1}{\sin\beta}\,[R_{j2}-i\gamma_5\cos\beta R_{j3}]\equiv a+i\tilde{a}\gamma_5. \qquad (3.61)
\end{aligned}
$$

Likewise, for the charged Higgs bosons [13]

$$
\begin{aligned}
H^+ b\bar{t}: &\qquad \frac{ig}{2\sqrt{2}m_W}\,[m_b(1+\gamma_5)\tan\beta+m_t(1-\gamma_5)\cot\beta], \\
H^- t\bar{b}: &\qquad \frac{ig}{2\sqrt{2}m_W}\,[m_b(1-\gamma_5)\tan\beta+m_t(1+\gamma_5)\cot\beta]. \qquad (3.62)
\end{aligned}
$$

The product of the $H_j t\bar{t}$ scalar and pseudoscalar couplings,

$$
\gamma_{CP}^{(j)} = -a\,\tilde{a} = \frac{\cos\beta}{\sin^2\beta}\,R_{j2}R_{j3} \qquad (3.63)
$$

plays an important role in determining the amount of CP violation in the top-quark sector.

As was seen in ref. [78], unless the Higgs boson is resonant with the $t\bar{t}$ system, CP violation is largest for small Higgs masses.

For the lightest Higgs boson, the coupling (3.61) becomes

$$
H_1 t\bar{t}: \qquad \frac{1}{\sin\beta}\,[\sin\alpha_1\cos\alpha_2-i\gamma_5\cos\beta\sin\alpha_2], \quad \text{with} \quad \gamma_{CP}^{(1)}=\frac{1}{2}\,\frac{\sin\alpha_1\sin(2\alpha_2)}{\tan\beta\sin\beta}, \qquad (3.64)
$$

where α_1 and α_2 are mixing angles of the Higgs mass matrix as defined by Eqs. (3.41) and (3.42). From (3.64), we see that low $\tan\beta$ are required for having large CP violation in the top-quark sector. However, according to (3.62), for low $\tan\beta$ the charged-Higgs Yukawa coupling is also enhanced.

In the next section we will turn to a different scenario, the SM-like Higgs bosons for the CP conserving case, and we will see how the 2HDM deviates from the SM.

3.7 SM-like Higgs Boson, CP Conserving case

Let us now consider a SM-like scenario, where the Higgs boson partial widths or coupling constants squared are assumed to be precisely measured, being in agreement with the

SM within the experimental accuracies. This can happen not only in the SM, but also if nature is described by some other theory, i.e., the 2HDM or MSSM.

The deviation of the 2HDM from the SM predictions can occur due to two sources: the mixing effect which appears at the tree level, and the quantum correction effect due to the loop contribution of the extra Higgs bosons. If the mixing between the CP-even Higgs bosons is large, hZZ coupling in the 2HDM significantly differs from the SM prediction at the tree level by the factor $\sin(\beta-\alpha)$, where α $(-\pi/2 < \alpha \leq 0)$ is the familiar mixing angle of the CP-even sector. Therefore one can obtain an indirect evidence of extended Higgs sector at the LHC [83]. On the other hand, hZZ coupling may be close to the SM prediction, i.e., in the SM-like regime where $\sin^2(\alpha-\beta) \simeq 1$ [75].

The presence of a SM-like Higgs boson is consistent with the 2HDM with parameters near the decoupling limit. The decoupling limit is also a regime in which all masses are very heavy and nearly mass-degenerate but the lightest Higgs boson mass is not. However, large Higgs masses (with significant mass splittings) can also arise in a non-decoupling parameter regime in which the λ_i are large. In this case, the heavy Higgs bosons are bounded from above by imposing unitarity constraints on the λ_i. These unitarity are obtained for the case of CP-conserving in [21], can be more severe in the CP-violating case [22, 84].

This scenario can be realized in two ways, depending on the value of the μ^2 parameter $(\mu^2 = v^2\nu = v^2 \mathrm{Re}\, m_{12}^2/2v_1v_2)$. For large μ^2 the additional Higgs bosons masses can be very large and almost degenerate, in such case there is decoupling of these heavy bosons from known particles, i.e., effects of these heavy particles disappear if their masses tend to infinity [85]. The non-decoupling case occurs at small μ^2, the large masses of such additional Higgs bosons arise from large quartic self-couplings (λ). On the other side there are constraints from the unitarity on the quartic coupling constants. These bounds force the heavy Higgs bosons to be lighter than 600 GeV [21]. In this scenario the additional Higgs bosons can be heavy enough to avoid direct observation even in the next generation of colliders, although some relevant effects can appear in the interaction of the lightest Higgs bosons [86, 87].

If the SM-like scenario is realized in the 2HDM, one needs to consider both possibilities: not only the light scalar Higgs boson, h, but also the heavier one, H. The ratios relative to the SM values, of the direct coupling constants of the Higgs bosons (h and H) to the gauge bosons $V = W$ or Z, to up and down quarks and to charged leptons (basic couplings) can be written as [75]

$$\begin{aligned}
\chi_V^h &= \sin(\beta-\alpha), \\
\chi_V^H &= \cos(\beta-\alpha), \\
\chi_u^h &= \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha), \\
\chi_u^H &= \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha), \\
\chi_d^h &= \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha), \\
\chi_u^H &= \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha),
\end{aligned} \tag{3.65}$$

and for the CP-odd Higgs boson A

$$\chi_V^A = 0, \quad \chi_u^A = -\cot\beta, \quad \chi_d^A = -\tan\beta \tag{3.66}$$

In the CP-conserving case, the trilinear coupling hH^+H^- takes the form

$$\chi^h_{H^\pm} = \big(1 - \frac{M_h^2}{2M_{H^\pm}^2}\big)\chi^h_V + \frac{M_h^2 - \mu^2}{2M_{H^\pm}^2}(\chi^h_u + \chi^h_d) \tag{3.67}$$

The contribution of the $H^\pm$ loop to $\Gamma_{\gamma\gamma}$ (the two-photon decay width) in the case of $\chi^h_j = 1$ is given by

$$|X_{\gamma\gamma}|^2 = 1 + \frac{M_h^2}{2M_{H^\pm}^2} - \frac{\mu^2}{M_{H^\pm}^2} \tag{3.68}$$

One can consider two cases of the SM-like Higgs bosons

- h as an SM-like Higgs boson. In this case h is the observed Higgs boson, with $\sin(\beta - \alpha) \approx \pm 1$ so $\cos(\beta - \alpha) \approx \pm 0$. For this solution H is heavy boson and can not be observed.

- H as an SM-like Higgs boson. In this case H is the observed Higgs boson, with $\cos(\beta - \alpha) \approx \pm 1$ so $\sin(\beta - \alpha) \approx \pm 0$. For this solution h boson can not be observed.

In the next chapter we will try to constrain the 2HDM theoretically by imposing the unitarity and positivity constraint.

Chapter 4

Theoretical Constraints on the 2HDM

The 2HDM presents a richer phenomenology due to the appearance of the charged and two more neutral Higgs particles. Many studies advocated to constrain the parameters of the 2HDM.

It is convenient to split the constraints into three categories:

(i) Theoretical consistency constraints: positivity of the potential [16, 17] and perturbative unitarity [20–22]. From the theoretical point of view, there are various consistency conditions. The potential has to be positive for large values of the fields. We also require the tree-level Higgs-Higgs scattering amplitudes to be unitary. Together, these constraints dramatically reduce the allowed parameter space of the model.

(ii) Experimental constraints on the charged-Higgs sector. These all come from B-physics, and are due to $b \to s\gamma$, B–$\bar{B}$ oscillations, and $B \to \tau\nu_\tau$. They are all independent of the neutral sector.

(iii) Experimental constraints on the neutral sector. These are predominantly due to the precise measurements of R_b, non-observation of a neutral Higgs boson at LEP2, $\Delta\rho$, and $a_\mu = \frac{1}{2}(g-2)_\mu$.

The first and third categories of constraints will depend on the neutral sector, i.e., the neutral Higgs masses and the mixing matrix. The second category is due to physical effects of the charged-Higgs Yukawa coupling in the B-physics sector. These are "general" in the sense that they do not depend on the spectrum of neutral Higgs bosons, i.e., they do not depend on the mixing (and possible CP violation) in the neutral sector.

When considering the different experimental constraints, our basic approach will be that they are all in agreement with the Standard Model, and simply let the experimental or theoretical uncertainty restrict possible 2HDM contributions (this procedure yields lower bounds on the charged-Higgs mass, possibly also other constraints). An alternative approach would be to actually fit the 2HDM to the data.

In this chapter we will study the theoretical constraints, and in the next chapter we will study the experimental constraints.

4.1 Positivity for CP non-conservation

Many studies have been devoted to investigate the positivity condition of the 2HDM potential, the Higgs potential should be bounded from below and be positive for large values of the fields [17, 18, 88–93]. At tree-level without explicit CP breaking, there is a minimum that preserves the $U(1)_{\rm em}$ and CP symmetries, that minimum is the global one [91]. According to Gunion and Haber [77], if one defines $a \equiv \Phi_1^\dagger\Phi_1$, $b \equiv \Phi_2^\dagger\Phi_2$, $c \equiv {\rm Re}\,\Phi_1^\dagger\Phi_2$ and $d \equiv {\rm Im}\,\Phi_1^\dagger\Phi_2$ we can rewrite the quartic term of the potential (3.20) with complex parameters as

$$\begin{aligned} V_4 &= \frac{1}{2}[\sqrt{\lambda_1}a - \sqrt{\lambda_2}b]^2 + [\lambda_3 + \sqrt{\lambda_1\lambda_2}](ab - c^2 - d^2) \\ &+2[\lambda_3 + \lambda_4 + \sqrt{\lambda_1\lambda_2}]c^2 + [{\rm Re}\,\lambda_5 - \lambda_3 - \lambda_4 - \sqrt{\lambda_1\lambda_2}](c^2 - d^2) \\ &-2cd{\rm Im}\,\lambda_5 + 2a[c{\rm Re}\,\lambda_6 - d{\rm Im}\,\lambda_6] + 2b[c{\rm Re}\,\lambda_7 - d{\rm Im}\,\lambda_7] \end{aligned} \tag{4.1}$$

For some special conditions of (4.1), one can get the following

- If $a \to \infty$ and $c = d = 0$ as a result of $V > 0$, then $\lambda_1 > 0$
- If $b \to \infty$ and $c = d = 0$ as a result of $V > 0$, then $\lambda_2 > 0$
- If $a \to \infty$, $b \to \infty$ and $c = d = 0$ as a result of $V > 0$, then $\lambda_3 > -\sqrt{\lambda_1\lambda_2}$
- A fourth condition arises by examining the direction in field space where $a\sqrt{\lambda_1} = b\sqrt{\lambda_2}$ and $ab = c^2 + d^2$. If one takes $c = \epsilon d$ (ϵ is very small parameter), $\lambda_6 = \lambda_7 = 0$ and requires the potential to be bounded from below for all ϵ, the polynomial in ϵ can be written as

$$\begin{aligned} V_4 &= d^2\Big[2\epsilon^2[{\rm Re}\,\lambda_5 + \lambda_3 + \lambda_4 + \sqrt{\lambda_1\lambda_2}] + \epsilon^2[{\rm Re}\,\lambda_5 - \lambda_3 - \lambda_4 - \sqrt{\lambda_1\lambda_2}] \\ &-[{\rm Re}\,\lambda_5 - \lambda_3 - \lambda_4 - \sqrt{\lambda_1\lambda_2} + 2\epsilon{\rm Re}\,\lambda_5]\Big] \end{aligned} \tag{4.2}$$

It easy to derive the condition $\lambda_3 + \lambda_4 \pm |\lambda_5| > -\sqrt{\lambda_1\lambda_2}$.

The above conditions are derived in another way in the appendix of [17].

In this section, we will try to write the two complex doublets of the Higgs field in the form

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_5 + i\phi_7 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_6 + i\phi_8 \end{pmatrix} \tag{4.3}$$

where ($\phi_i, i = 1, ..., 8$) are real fields. The Higgs fields can be reparameterized as

$$\begin{aligned} x_1 &= |\Phi_1|^2 = \phi_1^2 + \phi_2^2 + \phi_5^2 + \phi_7^2, \\ x_2 &= |\Phi_2|^2 = \phi_3^2 + \phi_4^2 + \phi_6^2 + \phi_8^2, \\ x_3 &= {\rm Re}\,(\Phi_1^\dagger\Phi_2) = \phi_1\phi_3 + \phi_2\phi_4 + \phi_5\phi_6 + \phi_7\phi_8, \\ x_4 &= {\rm Im}\,(\Phi_1^\dagger\Phi_2) = \phi_1\phi_4 - \phi_2\phi_3 + \phi_5\phi_8 - \phi_6\phi_7 \end{aligned} \tag{4.4}$$

Under CP transformation ($\Phi_1 \rightarrow \Phi_1^\star$, and $\Phi_2 \rightarrow \Phi_2^\star$) the invariants x_1, x_2, and x_3 remain the same but x_4 changes sign. The general 2HDM potential takes the form [91, 92]

$$\begin{aligned} V &= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{44}x_4^2 \\ &\quad + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 \end{aligned} \tag{4.5}$$

The potential (4.5) can be written in compact form as

$$V = A^T X + \frac{1}{2} X^T B X \tag{4.6}$$

where

$$A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}, \qquad B = \begin{pmatrix} 2b_{11} & b_{12} & b_{13} \\ b_{12} & 2b_{22} & b_{23} \\ b_{13} & b_{23} & 2b_{33} \end{pmatrix}, \qquad \text{and} \qquad X = \begin{pmatrix} x_1, & x_2, & x_3, & x_4 \end{pmatrix} \tag{4.7}$$

The 2HDM potential has three types of possible minima [8, 38]. In the first and second types of minima only neutral fields have vevs with two different possibilities. In one case only the two real fields ϕ_5 and ϕ_6 have vevs which explicitly breaks CP and is called N_1 minimum,

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{4.8}$$

and in the second case three fields ϕ_5, ϕ_6 and ϕ_7 have vevs which spontaneously breaks CP and is called N_2 minimum.

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1^{''} + i\delta \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} 0 \\ v_2^{''} \end{pmatrix}, \tag{4.9}$$

In the third type the fields ϕ_5, ϕ_6 and ϕ_3 have vevs which is called a charge breaking (CB) minimum. The vev ϕ_3 breaks the $U(1)_{\text{em}}$ symmetry and gives a mass to the photon.

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1^{'} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \alpha \\ v_2^{'} \end{pmatrix}, \tag{4.10}$$

It is easy to prove that if the normal minima exist, they are deeper than the charge breaking one [91]. At N_1 the non-zero vevs are $\phi_5 = v_1$ and $\phi_6 = v_2$, so that $x_1 = v_1^2$, $x_2 = v_2^2$, $x_3 = v_1 v_2$ and $x_4 = 0$. Let V' be a vector with components $V_i' = \partial V / \partial x_i$ and the value of the vector X at the minimum point is X_{N_1}. The potential at the minimum point N_1 takes the form

$$V_{N_1} = \frac{1}{2} A^T X_{N_1} = -\frac{1}{2} X_{N_1}^T B X_{N_1} \quad \Leftrightarrow X_{N_1}^T B X_{N_1} = -2V_{N_1} \tag{4.11}$$

The stationary conditions can be written as

$$\begin{aligned}
\frac{\partial V}{\partial v_1} &= 0, \quad \Leftrightarrow V_1'\frac{\partial x_1}{\partial v_1} + V_3'\frac{\partial x_3}{\partial v_1} = 0 \quad \Leftrightarrow V_1' = \big(-\frac{V_3'}{2v_1v_2}\big)v_2^2 \\
\frac{\partial V}{\partial v_2} &= 0, \quad \Leftrightarrow V_2'\frac{\partial x_2}{\partial v_2} + V_3'\frac{\partial x_3}{\partial v_2} = 0 \quad \Leftrightarrow V_2' = \big(-\frac{V_3'}{2v_1v_2}\big)v_1^2 \\
\frac{\partial V}{\partial \phi_7} &= 0, \quad \Leftrightarrow V_4'\frac{\partial x_4}{\partial \phi_7} = 0 \quad \Leftrightarrow V_4' = 0
\end{aligned} \tag{4.12}$$

therefore one can write V' as

$$V' = A + BX_{N_1} = \begin{pmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{pmatrix} = -\frac{V_3'}{2v_1v_2}\begin{pmatrix} v_2^2 \\ v_1^2 \\ -2v_1v_2 \\ 0 \end{pmatrix} \tag{4.13}$$

Also one can define Y as a vector with components $Y = (v_1'^2, v_2'^2 + \alpha^2, v_1'v_2', 0)$ and the stationary conditions can be written as

$$\begin{aligned}
\frac{\partial V}{\partial v_1'} &= 0, \quad \Leftrightarrow V_1'\frac{\partial x_1}{\partial v_1'} + V_3'\frac{\partial x_3}{\partial v_1'} = 0 \quad \Leftrightarrow V_1' = \big(-\frac{V_3'}{2v_1'v_2'}\big)v_2'^2 \\
\frac{\partial V}{\partial v_2'} &= 0, \quad \Leftrightarrow V_2'\frac{\partial x_2}{\partial v_2'} + V_3'\frac{\partial x_3}{\partial v_2'} = 0 \quad \Leftrightarrow V_2' = \big(-\frac{V_3'}{2v_1'v_2'}\big)v_1'^2 \\
\frac{\partial V}{\partial \phi_7} &= 0, \quad \Leftrightarrow V_4'\frac{\partial x_4}{\partial \phi_7} = 0 \quad \Leftrightarrow V_4' = 0 \\
\frac{\partial V}{\partial \alpha} &= 0, \quad \Leftrightarrow V_2'\frac{\partial x_2}{\partial \alpha} = 0 \quad \Leftrightarrow V_2' = 0
\end{aligned} \tag{4.14}$$

Also the vector V' can be rewritten as

$$V' = A + BY \tag{4.15}$$

The potential at a CB stationary point, $V_i' = 0$ is defined by

$$V_{CB} = \frac{1}{2}A^TY = -\frac{1}{2}Y^TBY \quad \Leftrightarrow Y^TBY = -2V_{CB} \tag{4.16}$$

Multiplying Eqs. (4.13) and (4.15) by $X_{N_1}^T$, one gets

$$X_{N_1}^TBY = X_{N_1}^TBX_{N_1} = -2V_{N_1} \tag{4.17}$$

since the matrix B is symmetric, therefore

$$X_{N_1}^TBY = Y^TBX_{N_1} = -2V_{N_1} \tag{4.18}$$

From the above Eqs. (4.11), (4.16), (4.18) one can write

$$Y^TV' = -Y^TBY + Y^TBX_{N_1} \tag{4.19}$$

or

$$
\begin{aligned}
V_{CB} - V_{N_1} &= \frac{1}{2} Y^T V' \\
&= \frac{M_{H^\pm}^2}{2v^2} [(v_1' v_2 - v_2' v_1)^2 + \alpha^2 v_1^2]
\end{aligned} \tag{4.20}
$$

The right hand side of (4.20) is a sum of squares, so it must be positive, therefore $V_{CB} - V_{N_1} > 0$. It means that the charge breaking stationary point, if it exists, is always located above the N_1 minimum. The stationary point is a saddle point which means it can not be the global minimum point.

In the next section we will discuss the unitarity constraints on the 2HDM parameter space for the CP-conserving case.

4.2 Unitarity constraint on 2HDM for CP-conservation

To constrain the scalar potential parameters of the 2HDM one can demand that the tree-level unitarity is preserved in all different scattering processes: scalar-scalar scattering, gauge boson-gauge boson scattering and scalar-gauge boson scattering [94]. The unitarity condition plays an important role to put upper bounds for the neutral as well as charged Higgs boson mass in the 2HDM. The mass of the Higgs boson which is proportional to the Higgs quartic coupling λ, may be bounded from above, provided that the quartic coupling is not so large as to violate the validity of perturbative calculations [20, 95].

Maalampi et al. [96] derived an upper bound of the neutral Higgs boson mass of the 2HDM by numerical analysis, which gave them more or less the same bound as Lee, Quigg and Thacker (LQT) [95]. But they did not consider a broad class of scattering processes to derive constraints on all of the charged and neutral Higgs boson masses. The self interactions of the scalars as well as the longitudinal gauge fields interactions become strong as the Higgs mass m_H increases. Lee, Quigg and Thacker showed [95] that when m_H exceeds a certain critical value

$$
m_H \geq M_{LQT} \equiv \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} \simeq 1 \text{ TeV} \tag{4.21}
$$

the elastic s-wave scattering of the longitudinal vector bosons at high energy, $s \gg m_H^2$, violates unitarity at tree-level. Above this critical value perturbation theory will be no longer valid. The partial wave amplitude $|a_l(s)|$ for scattering of two spin-0 particles is defined as

$$
a_l(s) = \frac{1}{32\pi} \int_{-1}^{1} d\cos\theta P_l(\cos\theta) T(s,t,u) \tag{4.22}
$$

where P_l are Legendre polynomials, θ is the scattering angle and T is the scattering amplitude. The partial wave amplitude satisfies [97]

$$
\operatorname{Im} a_l \geq |a_l|^2 \tag{4.23}
$$

for the scattering of massless particles. We can rewrite (4.23) as

$$
\operatorname{Im} a_l \geq (\operatorname{Re} a_l)^2 + (\operatorname{Im} a_l)^2 \tag{4.24}
$$

or

$$(\mathrm{Re}\, a_l)^2 \leq \mathrm{Im}\, a_l(1 - \mathrm{Im}\, a_l) \tag{4.25}$$

The right-hand side of (4.25) is bounded by 1/4, it implies

$$|\mathrm{Re}\, a_l| \leq \frac{1}{2} \tag{4.26}$$

where 1/2 is the radius of the Argand circle. Since $\mathrm{Im}\, a_l \leq |a_l|$ (Schwartz inequality), Eq.(4.23) implies

$$|a_l(s)| \leq 1 \tag{4.27}$$

for all l, which is the normal unitarity condition for the scattering matrix at energy scale s [20].

One can use the perturbation calculations up to a cut-off scale $s_c > m_H^2$ determined from the condition ($\infty > s_c > m_H^2$)

$$|a_0(s_c)| = 1 \tag{4.28}$$

If $a_0(s) \geq 1$ for $s > m_H^2$, one can not apply the perturbation theory. To derive the unitarity constraints on the scalar masses or quartic coupling $\lambda' s$, one should consider the following

- At very high energy collisions, the dominant contribution to the amplitude of the two-body scattering $S_1S_2 \to S_3S_4$ is the one which is mediated by the quartic coupling and the Higgs-Higgs scattering matrix at high enough energy at tree-level contains only s-wave ($J = 0$) amplitudes. The tree-level unitarity constraints require that the eigenvalues of this scattering matrix be less than the unitarity limit. Violation of the tree-level unitarity constraints implies that the tree-level calculations are no more reliable and do not respect the physics of the model [84]. The coefficients of the scattering matrix at high energy are given only by parameters λ_i of the Higgs potential, the unitarity constraints can be written in the limitation of $\lambda' s$ i.e., $\lambda < 16\pi/3$ [98]. To derive the unitarity constraints, one should construct the scattering matrix for all the physical Higgs states in the tree-level approximation at high enough energy and diagonalize it.

- Feynman diagram containing triple Higgs couplings are suppressed in energy on the dimensional account. Therefore the unitarity constraint $|a_0| \leq 1/2$ reduces to the following constraint on the quartic coupling, $|Q(S_1S_2S_3S_4)| \leq 8\pi$, where Q is the four point vertex coupling constant for the scattering process $S_1S_2 \to S_3S_4$.

- The quartic vertices written in terms of physical fields $H^\pm$, $G^\pm$, h^0, H^0, A^0 and G^0 are complicated functions of λ_i, α and β. This problem can be solved according to [20] by using the fact that the S-matrix expressed in terms of physical fields can be transformed into an S-matrix in terms of non-physical fields $\phi_i^\pm$, η_i and χ_i by making a unitary transformation. It is easy to compute the S-matrix of non-physical fields from the Higgs potential i.e., see Eq. (3.39). Therefore the full set of the scalar scattering processes can be expressed as an S-matrix composed of 4 submatrices which do not couple with each other due to charge conservation and CP-invariance [21].

The first submatrix corresponding to the scatterings of the following states: ($\phi_1^+\phi_2^-$, $\phi_2^+\phi_1^-$, $\eta_1\chi_2$,$\eta_2\chi_1$ $\chi_1\chi_2$, $\eta_1\eta_2$) is a 6×6 matrix with eigenvalues [20, 21, 84, 94]

$$\begin{aligned}
e_1 &= \lambda_3 + 2\lambda_4 - 3(\mathrm{Re}\,\lambda_5) \\
e_2 &= \lambda_3 - \mathrm{Re}\,\lambda_5 \\
f_1 &= f_2 = \lambda_3 + \lambda_4 \\
f_+ &= \lambda_3 + 2\lambda_4 + 3(\mathrm{Re}\,\lambda_5) \\
f_- &= \lambda_3 + \mathrm{Re}\,\lambda_5
\end{aligned} \tag{4.29}$$

The second submatrix corresponding to the scatterings of the following states: ($\phi_1^+\phi_1^-$, $\phi_2^+\phi_2^-$, $\frac{\chi_1\chi_1}{\sqrt{2}}$, $\frac{\chi_2\chi_2}{\sqrt{2}}$, $\frac{\eta_1\eta_1}{\sqrt{2}}$, $\frac{\eta_2\eta_2}{\sqrt{2}}$) is a 6×6 matrix with eigenvalues

$$\begin{aligned}
a_\pm &= \frac{1}{2}\left[3(\lambda_1+2\lambda_2) \pm \sqrt{9(\lambda_1-\lambda_2)^2 + 4(2\lambda_3+\lambda_4)^2}\right] \\
b_\pm &= \frac{1}{2}\left[(\lambda_1+2\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\right] \\
c_\pm &= \frac{1}{2}\left[(\lambda_1+2\lambda_2) \pm \sqrt{(\lambda_1-\lambda_2)^2 + 4(\mathrm{Re}\,\lambda_5)^2}\right]
\end{aligned} \tag{4.30}$$

The third submatrix, is a 2×2 matrix and corresponds to the basis ($\eta_1\chi_1$,$\eta_2\chi_2$) with eigenvalues $d_\pm = c_\pm$.

Akeroyd et al. [21] confirm results of Kanemura et al. [20] and they also add the two body scattering between the 8 charged states: $\eta_1\phi_1^+$, $\eta_2\phi_1^+$, $\chi_1\phi_1^+$, $\chi_2\phi_1^+$, $\eta_1\phi_2^+$, $\eta_2\phi_2^+$, $\chi_1\phi_2^+$, $\chi_2\phi_2^+$. The fourth submatrix is 8×8 with the following eigenvalues, f_-, e_2, f_1, $c_\pm$, $b_\pm$ and p_1, where the new eigenvalue due to the scattering between the 8 charged states can be written as

$$p_1 = \lambda_3 - \lambda_4 \tag{4.31}$$

All the eigenvalues mentioned above are constrained as follows:

$$|a_\pm|, \quad |b_\pm|, \quad |c_\pm|, \quad |d_\pm|, \quad |f_\pm|, \quad |e_{1,2}|, \quad |f_{1,2}|, \quad |p_1| \le 8\pi \tag{4.32}$$

In the next section we will try to discuss the case of CP-violation for hard Z_2 violation.

4.3 Unitarity constrain on 2HDM for CP-violation

Ginzburg et al [84] derived the eigenvalues of all scattering processes that are derived in sec. 4.2 by another way, using the fact that at high energy the total weak isospin σ and the total hypercharge Y, are conserved with possible values $\sigma = 0, 1$ and $Y = 0, 2, -2$. Terms in the 2HDM potential $(\phi_a^*\phi_b)(\phi_c^*\phi_d)$ induce transitions in all possible cross-channels, i.e., $\phi_a\phi_b^* \to \phi_c^*\phi_d$, $\phi_a\phi_d^* \to \phi_c^*\phi_b$, $\phi_a\phi_c \to \phi_b\phi_d$, where a, b, c and d indices are the components of the 2HDM doublets. In order to calculate the amplitude of the transition of the initial two-Higgs state $(\phi\phi)^{in}$ to a final state $(\phi\phi)^f$ one needs to rewrite the potential in such way $(\phi\phi)^{in} \times (\phi\phi)^f$. The coefficient in front of this product gives the amplitude of the transition.

The scattering matrix in the tree approximation for each state with certain quantum numbers Y and σ is given by

$$S_{Y,\sigma} = \langle (\phi\phi)^{f}_{Y,\sigma} | \hat{S} | (\phi\phi)^{in}_{Y,\sigma} \rangle \tag{4.33}$$

These scattering matrices can be written in terms of $\lambda' s$ according to Ginzburg et al [84] as

$$S_{Y=2,\sigma=1} = \frac{1}{16\pi}\begin{pmatrix} \lambda_1 & \lambda_5 & \sqrt{2}\lambda_6 \\ \lambda_5^* & \lambda_2 & \sqrt{2}\lambda_7^* \\ \sqrt{2}\lambda_6^* & \sqrt{2}\lambda_7 & \lambda_3+\lambda_4 \end{pmatrix}, \tag{4.34}$$

$$S_{Y=2,\sigma=0} = \frac{1}{16\pi}(\lambda_3 - \lambda_4), \tag{4.35}$$

$$S_{Y=0,\sigma=1} = \frac{1}{16\pi}\begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 \end{pmatrix}, \tag{4.36}$$

$$S_{Y=0,\sigma=0} = \frac{1}{16\pi}\begin{pmatrix} 3\lambda_1 & 2\lambda_3+\lambda_4 & 3\lambda_6 & 3\lambda_6^* \\ 2\lambda_3+\lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3+2\lambda_4 & 3\lambda_5^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3+2\lambda_4 \end{pmatrix}, \tag{4.37}$$

$$\tag{4.38}$$

The unitarity condition is $S < 1$, it means that each eigenvalue of the matrix is less than 8π. By diagonalizing the 2×2 off-diagonal submatrix at the corners of the scattering amplitude matrices, one can obtain the eigenvalues derived by Akeroyd et al. [21], which are the necessary conditions for unitarity. The terms describing the hard violation of Z_2 symmetry λ_6 and λ_7 modify the eigenvalues [84].

It is important to write $\lambda' s$ in explicit form in terms of the rotation matrix, the neutral mass eigenvalues, μ^2 and $M_{H^\pm}$, to study the unitarity constraints on the 2HDM. These can be derived from the mass matrix equations (see, for example, [17], [18]) as:

$$\lambda_1 = \frac{1}{c_\beta^2 v^2}[c_1^2 c_2^2 M_1^2 + (c_1 s_2 s_3 + s_1 c_3)^2 M_2^2 + (c_1 s_2 c_3 - s_1 s_3)^2 M_3^2 - s_\beta^2 \mu^2], \tag{4.39}$$

$$\lambda_2 = \frac{1}{s_\beta^2 v^2}[s_1^2 c_2^2 M_1^2 + (c_1 c_3 - s_1 s_2 s_3)^2 M_2^2 + (c_1 s_3 + s_1 s_2 c_3)^2 M_3^2 - c_\beta^2 \mu^2], \tag{4.40}$$

$$\lambda_3 = \frac{1}{c_\beta s_\beta v^2}\{c_1 s_1[c_2^2 M_1^2 + (s_2^2 s_3^2 - c_3^2) M_2^2 + (s_2^2 c_3^2 - s_3^2) M_3^2] + s_2 c_3 s_3 (c_1^2 - s_1^2)(M_3^2 - M_2^2)\} + \frac{1}{v^2}[2M_{H^\pm}^2 - \mu^2], \tag{4.41}$$

$$\lambda_4 = \frac{1}{v^2}[s_2^2 M_1^2 + c_2^2 s_3^2 M_2^2 + c_2^2 c_3^2 M_3^2 + \mu^2 - 2M_{H^\pm}^2], \tag{4.42}$$

$$\mathrm{Re}\,\lambda_5 = \frac{1}{v^2}[-s_2^2 M_1^2 - c_2^2 s_3^2 M_2^2 - c_2^2 c_3^2 M_3^2 + \mu^2], \tag{4.43}$$

$$\mathrm{Im}\,\lambda_5 = \frac{-1}{c_\beta s_\beta v^2}\{c_\beta[c_1 c_2 s_2 M_1^2 - c_2 s_3 (c_1 s_2 s_3 + s_1 c_3) M_2^2 + c_2 c_3 (s_1 s_3 - c_1 s_2 c_3) M_3^2] + s_\beta[s_1 c_2 s_2 M_1^2 + c_2 s_3 (c_1 c_3 - s_1 s_2 s_3) M_2^2 - c_2 c_3 (c_1 s_3 + s_1 s_2 c_3) M_3^2]\}, \tag{4.44}$$

These equations are the analogues of those of [99] for the CP-conserving 2HDM and the squared-mass of the heay Higgs boson can be experessed in terms of M_1^2, M_2^2, R and $\tan\beta$ as

$$M_3^2 = \frac{M_1^2 R_{13}(R_{12}\tan\beta - R_{11}) + M_2^2 R_{23}(R_{22}\tan\beta - R_{21})}{R_{33}(R_{31} - R_{32}\tan\beta)} \tag{4.45}$$

The upper bounds on the neutral as well as charged Higgs boson masses are derived by assuming all the quartic coupling (4.39)-(4.44) are perturbative.

In the next chapter we will study the experimental constraints on the charged-Higgs sector of the 2HDM.

Chapter 5

Experimental constraints on the charged-Higgs sector

The SM prediction of the muon anomalous magnetic moment a_μ or $(g-2)$ deviates from the present experimental value by $2-3\ \sigma$ [100]. Many extensions of the SM are capable of giving rise to such deviation. Theorists try to impose constraints on the parameter space for specific models not only to exclude part of the parameter space but also to predict where the model is valid.

Several experimental constraints restrict the 2HDM, such as $B-\bar{B}$ oscillations, the partial decay width R_b for the process $Z \to b\bar{b}$, the precision measurements of the parameter $\Delta\rho$ which measures the deviation of the W and Z self-energies from the standard model value, a_μ, $\bar{B} \to \tau\bar{\nu}_\tau$, $\bar{B} \to X_s\gamma$, and precision measurements of the $W^\pm$ boson properties and Higgs boson mass at the Large Electron-Positron collider (LEP2). The $B-\bar{B}$ oscillations and branching ratio R_b exclude low values of $\tan\beta$, whereas the $\bar{B} \to X_s\gamma$ rate excludes low values of the charged-Higgs mass, $M_{H^\pm}$. The precise measurements at LEP of the ρ parameter constrain the mass splitting in the Higgs sector, and force the masses to be not far from the Z mass scale [67]. We divided the experimental constraints on the 2HDM into two groups as

- Experimental constraints on the charged-Higgs sector. These all come from B-physics, such as $\bar{B} \to X_s\gamma$, B–$\bar{B}$ oscillations, and $\bar{B} \to \tau\bar{\nu}_\tau$. They are all independent of the neutral sector. These constraints will be discussed in this chapter.

- Experimental constraints on the neutral sector. These are predominantly due to the precise measurements of R_b, non-observation of a neutral Higgs boson at LEP2, $\Delta\rho$, and $a_\mu = \frac{1}{2}(g-2)_\mu$. These constraints will be discussed in the next chapter.

In the next section we will start to discuss the experimental constraints on the 2HDM by $\bar{B} \to X_s\gamma$.

5.1 The $\bar{B} \to X_s\gamma$ Constraint on the 2HDM

The inclusive decay $\bar{B} \to X_s\gamma$ is described at the partonic level by the weak decay $b \to s\gamma$, corrected for short-distance QCD effects. The perturpative QCD corrections are important in this decay, enhancing the rate by 2-3 times, which make the theoretical prediction compatible with the experimental rate within the error [101]. The perturbative QCD corrections introduce large logarithms $\alpha_s^n(\mu)\log^m(\mu/M)$, $(m \leq n)$, where $\alpha_s = g^2/4\pi$ (g is the strong coupling), M is a large scale ($M = m_t$ or m_W) and μ is the renormalization scale. By using renormalization group equations (RGE) the large logarithms are resummed by evolving the Wilson coefficients $C_i(\mu)$ from $\mu = \mu_W = m_W$ to $\mu = \mu_b \simeq m_b$ [102, 103].

5.1.1 The $\bar{B} \to X_s\gamma$ Constraint at leading order (LO)

The weak decay $b \to s\gamma$ is calculated at the LO, which is at order α_s^0. The strongest bound on the charged Higgs mass in the 2HDM type-II comes from the B decays i.e., the inclusive decay $b \to s\gamma$. This branching ratio is important because it is sensitive to the physics beyond the Fermi scale at the leading order (LO) so that it is a good probe for new physic. It is strongly enhanced by QCD corrections, also the non-perturbative corrections to the inclusive decay is small [104, 105].

The effective Lagrangian can be written as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i(\mu) \tag{5.1}$$

where G_F is the Fermi constant, V_{ij} are elements of the CKM matrix, $Q_i(\mu)$ are the relevant operators and $C_i(\mu)$ are the corresponding Wilson coefficients. The operators $Q_i(\mu)$ can be defined as

$$\begin{aligned}
Q_1 &= (\bar{s}\gamma_\mu T^a P_L c)(\bar{c}\gamma^\mu T^a P_L b), & Q_2 &= (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b),\\
Q_3 &= (\bar{s}\gamma_\mu P_L b)\sum_q(\bar{q}\gamma^\mu q), & Q_4 &= (\bar{s}\gamma_\mu T^a P_L b)\sum_q(\bar{q}\gamma^\mu T^a q),\\
Q_5 &= (\bar{s}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} P_L b)\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q), & Q_6 &= (\bar{s}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a P_L b)\sum_q(\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q),\\
Q_7 &= \frac{e}{16\pi^2}\big[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)b\big]F_{\mu\nu}, & Q_8 &= \frac{g_s}{16\pi^2}\big[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)T^a b\big]G^a_{\mu\nu},
\end{aligned} \tag{5.2}$$

where $T^a (a = 1, \ldots, 8)$ are $SU(3)$ color generators, g_s and e are the strong and electromagnetic coupling constants and $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. The Q_1 and Q_2 are the current-current operators, $Q_3 - Q_6$ are QCD penguin operators and the magnetic penguin operators for $b \to s\gamma$ and $b \to sg$ are Q_7 and Q_8 respectively.

It is convenient to use linear combinations of the Wilson coefficient which are called "effective coefficients" [106]

$$C_i^{\rm eff}(\mu) = \begin{cases} C_i(\mu) & \text{for } i = 1, \ldots, 6; \\ C_7(\mu) + \sum_{i=1}^{6} y_i C_i(\mu) & \text{for } i = 7; \\ C_8(\mu) + \sum_{i=1}^{6} z_i C_i(\mu) & \text{for } i = 8. \end{cases} \tag{5.3}$$

where the numbers y_i and z_i are defined respectively in the minimum subtraction ($\overline{\rm MS}$) scheme as $y = (0, 0, -\frac{1}{3}, -\frac{4}{9}, -\frac{20}{3}, -\frac{80}{9})$ and $z = (0, 0, 1, -\frac{1}{6}, 20, -\frac{10}{3})$.

The leading-order contributions to the effective coefficients $C_i^{\text{eff}}(\mu)$ are regularization- and renormalization-scheme independent, which is not true for the original coefficients $C_7(\mu)$ and $C_8(\mu)$. The effective coefficients evolve according to their renormalization Group Equation (RGE) driven by the anomalous dimension matrix $\hat{\gamma}^{\text{eff}}$,

$$\mu \frac{d}{d\mu} C_i^{\text{eff}}(\mu) = C_j^{\text{eff}}(\mu) \hat{\gamma}^{\text{eff}}(\mu) \tag{5.4}$$

One expands the $\hat{\gamma}^{\text{eff}}$ matrix perturbatively as follows

$$\hat{\gamma}^{\text{eff}} = \frac{\alpha_s}{4\pi} \hat{\gamma}^{(0)\text{eff}} + \frac{\alpha_s^2}{(4\pi)^2} \hat{\gamma}^{(1)\text{eff}} + \ldots \tag{5.5}$$

where $\hat{\gamma}^{(0)\text{eff}}$ and $\hat{\gamma}^{(1)\text{eff}}$ are given in [106].

The effective Wilson coefficients can be expanded in powers of α_s as follows

$$C_i^{\text{eff}}(\mu) = C_i^{(0)\text{eff}} + \frac{\alpha_s}{4\pi} C_i^{(1)\text{eff}} + \ldots \tag{5.6}$$

The effective Wilson coefficients at scale $\mu_W = m_W$, at the leading order are defined as [107–109]

$$C_{i(\text{SM})}^{(0)\text{eff}}(\mu_W) = C_i^{(0)}(\mu_W) = \begin{cases} 0 & \text{for } i = 1, 3, 4, 5, 6; \\ 1 & \text{for } i = 2; \\ \frac{3x^3 - 2x^2}{4(x-1)^4} \ln x + \frac{-8x^3 - 5x^2 + 7x}{24(x-1)^3} & \text{for } i = 7; \\ \frac{-3x^2}{4(x-1)^4} \ln x + \frac{-x^3 + 5x^2 + 2x}{8(x-1)^3} & \text{for } i = 8 \end{cases} \tag{5.7}$$

where

$$x = \frac{m_{t\overline{\text{MS}}}^2(\mu_W)}{m_W^2} \tag{5.8}$$

The branching ratio suffers from a large theoretical uncertainty, $\sim 30 - 40\%$ coming mainly from the choices of the renormalization and matching scale, which is not well defined at the LO. At leading logarithmic order (LO), the branching ratio is given by [110–112]:

$$\mathcal{B}(\bar{B} \to X_s\gamma) = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{\text{e.m.}}}{\pi g(z)} |C_7^{(0)\text{eff}}(\mu_b)|^2 \times \mathcal{B}(\bar{B} \to X_c e \bar{\nu}_e), \tag{5.9}$$

where the first factor is a ratio of CKM matrix elements, $g(z = m_c^2/m_b^2)$ is a phase space factor, $g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln(z)$. The effective Wilson coefficients for the SM at the scale $\mu = \mu_b$ are defined as [109, 113]

$$C_1^{(0)\text{eff}}(\mu_b) = \left(\eta^{\frac{6}{23}} - \eta^{\frac{-12}{23}}\right) C_2^{(0)\text{eff}}(\mu_W) + \sum_{i=1}^{5} h_i \eta^{a_i} C_2^{(0)\text{eff}}(\mu_W) \tag{5.10}$$

$$C_2^{(0)\text{eff}}(\mu_b) = \left(\frac{2}{3}\eta^{\frac{6}{23}} + \frac{1}{3}\eta^{\frac{-12}{23}}\right) C_2^{(0)\text{eff}}(\mu_W) + \sum_{i=1}^{5} h_i \eta^{a_i} C_2^{(0)\text{eff}}(\mu_W) \tag{5.11}$$

$$C_7^{(0)\mathrm{eff}}(\mu_b) = \eta^{\frac{16}{23}} C_7^{(0)\mathrm{eff}}(\mu_W) + \frac{8}{3}(\eta^{\frac{14}{23}} - \eta^{\frac{16}{23}}) C_8^{(0)\mathrm{eff}}(\mu_W) + C_2^{(0)\mathrm{eff}}(\mu_W) \sum_{i=1}^{8} h_i \eta^{a_i} \tag{5.12}$$

$$C_8^{(0)\mathrm{eff}}(\mu_b) = \eta^{\frac{14}{23}} C_8^{(0)\mathrm{eff}}(\mu_W) + \sum_{i=1}^{5} h_i \eta^{a_i} C_2^{(0)\mathrm{eff}}(\mu_W) \tag{5.13}$$

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b)$. The explicit values of $a_i's$ and $h_i's$ are given by [109, 114]

$$\begin{aligned} a_i &= \Big(\frac{14}{23}, \frac{16}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456\Big) \\ h_i &= \Big(\frac{626126}{272277}, -\frac{56281}{51730}, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057\Big) \end{aligned} \tag{5.14}$$

The charged Higgs particle gives a new contribution to the Wilson coefficients of the effective theory and they can be computed at the matching scale μ_W, since they contain all the relevant ultraviolet information. The interaction between quarks and the charged Higgs field $H^{\pm}$ is defined by the Lagrangian [115]

$$\mathcal{L} = (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^{3} \bar{u}\Big(\cot\beta m_{u_i} V_{ij} \frac{1-\gamma_5}{2} + \tan\beta V_{ij} m_{d_i} \frac{1+\gamma_5}{2}\Big) d_j H^{\pm} + \mathrm{h.c.}, \tag{5.15}$$

where i, j are generation indices, $m_{u,d}$ the quark masses.

The coefficients $C_i(\mu_W)$ at the leading order of the 2HDM are given by [113, 116]

$$C_{i(\mathrm{2HDM})}^{(0)\mathrm{eff}}(\mu_W) = C_{i(\mathrm{2HDM})}^{(0)}(\mu_W) = \begin{cases} 0 & \text{for } i = 1,3,4,5,6; \\ 1 & \text{for } i = 2; \\ C_{7(\mathrm{SM})}^{(0)\mathrm{eff}}(\mu_W) - \frac{A(y_t)}{6}\cot^2\beta - B(y_t); & \text{for } i = 7; \\ C_{8(\mathrm{SM})}^{(0)\mathrm{eff}}(\mu_W) - \frac{D(y_t)}{6}\cot^2\beta - E(y_t) & \text{for } i = 8 \end{cases} \tag{5.16}$$

where $x_t = m_t^2/m_W^2$ and $y_t = m_t^2/m_{H^{\pm}}^2$ and the Inami-Lim functions are given by [107]

$$A(y) = y\Big[\frac{8y^2+5y-7}{12(y-1)^3} - \frac{(3y^2-2y)\ln y}{2(y-1)^4}\Big]; \tag{5.17}$$

$$B(y) = y\Big[\frac{5y-3}{12(y-1)^2} - \frac{(3y-2)\ln y}{6(y-1)^3}\Big]; \tag{5.18}$$

$$D(y) = y\Big[\frac{y^2-5y-2}{4(y-1)^3} + \frac{3y\ln y}{2(y-1)^4}\Big]; \tag{5.19}$$

$$E(y) = y\Big[\frac{y-3}{4(y-1)^2} + \frac{\ln y}{2(y-1)^3}\Big] \tag{5.20}$$

where $A(y)$, $B(y)$, $D(y)$ and $E(y)$ are coming from the contributions of the 2HDM at the LO.

In the next subsection we will try to see the effects of the two loop caculations and QCD effects at the Next to Leading Order (NLO) on the branching ratio $\Gamma(\bar{B} \to X_s\gamma)$.

5.1.2 The $\bar{B} \to X_s\gamma$ Constraint at Next to Leading Order

The two-loop matching condition, needed for a complete NLO calculation, was first obtained by Adel et al. [117] and later confirmed in [115,118,119], using different techniques. The two-loop corrections and the determination of the $\mathcal{O}(\alpha_s)$ elements of the anomalous dimension matrix are calculated in [106, 120]. The NLO of the 2HDM are calculated by Ciuchini et al. [115] and Borzumati et al. [113]. At the NLO, the sensitivity of the branching ratio to the scale is however significantly reduced [106,115,117,121–126]. Also it has been found that the NLO effects weaken the constraints on the allowed region in the $\tan\beta$–$M_{H^\pm}$ plane [109,113], the bound on $M_{H^\pm}$ is significantly relaxed.

The NLO top-quark running mass at the scale $\mu = m_W$ is given by [115]

$$m_t(\mu_W) = m_t(m_t)\Big[\frac{\alpha_s(\mu_W)}{\alpha_s(m_t)}\Big]^{\gamma_0^m/2\beta_0} \times \Big[1 + \frac{\alpha_s(m_t)}{4\pi}\frac{\gamma_0^m}{2\beta_0}\Big(\frac{\gamma_1^m}{\gamma_0^m} - \frac{\beta_1}{\beta_0}\Big)\Big(\frac{\alpha_s(\mu_W)}{\alpha_s(m_t)} - 1\Big)\Big] \tag{5.21}$$

where

$$\beta_0 = 11 - \frac{2}{3}n_f, \quad \beta_1 = 102 - \frac{38}{3}n_f \tag{5.22}$$

$$\gamma_0^m = 8, \quad \gamma_1^m = \frac{404}{3} - \frac{40}{9}n_f \tag{5.23}$$

where n_f is the effective number of quark flavours at the LO. The NLO corrections to the Wilson coefficient for the SM at the scale $\mu = \mu_W = m_W$, can be summarized as [113,115]

$$C^{(1)\mathrm{eff}}_{i(\mathrm{SM})}(\mu_W) = \begin{cases} 15 + 6\ln\frac{\mu_W^2}{m_W^2} & \text{for } i = 1; \\ 0 & \text{for } i = 2,3,5,6; \\ E_0(x) + \frac{2}{3}\ln\frac{\mu_W^2}{m_W^2} & \text{for } i = 4; \\ W_{7(\mathrm{SM})} + M_{7(\mathrm{SM})} & \text{for } i = 7 \\ W_{8(\mathrm{SM})} + M_{8(\mathrm{SM})} & \text{for } i = 8 \end{cases} \tag{5.24}$$

where

$$E_0(x) = \frac{x(x^2+11x-18)}{12(x-1)^3} + \frac{x^2(4x^2-16x+15)}{6(x-1)^4}\ln x - \frac{2}{3}\ln x - \frac{2}{3}, \tag{5.25}$$

$$\begin{aligned} W_{7(\mathrm{SM})} &= \frac{-16x^4-122x^3+80x^2-8x}{9(x-1)^4}Li_2\big(1-\frac{1}{x}\big) + \frac{6x^4+46x^3-28x^2}{3(x-1)^5}\ln^2 x \\ &+ \frac{-102x^5-588x^4-2262x^3+3244x^2-1364x+208}{81(x-1)^5}\ln x \\ &+ \frac{1646x^4+12250x^3-10740x^2+2509x-436}{486(x-1)^4}, \end{aligned} \tag{5.26}$$

$$\begin{aligned} W_{8(\mathrm{SM})} &= \frac{-4x^4+40x^3+41x^2+x}{6(x-1)^4}Li_2\big(1-\frac{1}{x}\big) + \frac{-17x^3-31x^2}{2(x-1)^5}\ln^2 x \\ &+ \frac{-210x^5+1086x^4+4893x^3+2857x^2-1994x+208}{216(x-1)^5}\ln x \\ &+ \frac{737x^4-14102x^3-28209x^2+610x-508}{1296(x-1)^4}, \end{aligned} \tag{5.27}$$

$$
\begin{aligned}
M_{7(\mathrm{SM})} &= \frac{82x^5+301x^4+703x^3-2197x^2+1319x-208}{81(x-1)^5} \\
&\quad -\frac{(162x^4+1242x^3-756x^2)\ln x}{81(x-1)^5},
\end{aligned}
\tag{5.28}
$$

$$
\begin{aligned}
M_{8(\mathrm{SM})} &= \frac{77x^5-475x^4-1111x^3+607x^2+1042x-140}{108(x-1)^5} \\
&\quad +\frac{(918x^3+1674x^2)\ln x}{81(x-1)^5}
\end{aligned}
\tag{5.29}
$$

where $Li_2(z)$ is the dilog function and is defined as

$$
Li_2(z) = -\int_0^z \frac{dt}{t}\ln(1-t) \tag{5.30}
$$

At the NLO, the Wilson coefficients for the 2HDM at the scale $\mu = \mu_W = m_W$ can be written as [113, 115]

$$
C^{(1)\mathrm{eff}}_{i(2\mathrm{HDM})}(\mu_W) = \begin{cases}
15+6\ln\frac{\mu_W^2}{m_W^2} & \text{for } i=1; \\
0 & \text{for } i=2,3,5,6; \\
E_0(x)+\frac{2}{3}\ln\frac{\mu_W^2}{m_W^2}+E_H(x)\cot^2\beta & \text{for } i=4; \\
C^{(1)\mathrm{eff}}_{7(\mathrm{SM})}+\cot^2\beta(G_7^H(y)+N_7^H(y))+M_7^H(y)+Z_7^H(y) & \text{for } i=7; \\
C^{(1)\mathrm{eff}}_{8(\mathrm{SM})}+\cot^2\beta(G_8^H(y)+N_8^H(y))+M_8^H(y)+Z_8^H(y) & \text{for } i=8
\end{cases}
\tag{5.31}
$$

where

$$
E_H = \frac{y}{36}\Big[\frac{7y^2-36y^2+45y-16+(18y-12)\ln y}{(y-1)^4}\Big], \tag{5.32}
$$

$$
\begin{aligned}
G_7^H(y) &= \frac{2y}{9}\Big[\frac{8y^3-37y^2+18y}{(y-1)^4}Li_2\big(1-\frac{1}{y}\big)+\frac{3y^3+23y^2-14y}{(y-1)^5}\ln^2 y \\
&\quad +\frac{21y^4-192y^3-174y^2+251y-50}{9(y-1)^5}\ln y \\
&\quad +\frac{-1202y^3+7569y^2-5436y+797}{108(y-1)^4}\Big]-\frac{4}{9}E_H,
\end{aligned}
\tag{5.33}
$$

$$
N_7^H(y) = \frac{y}{27}\Big[\frac{-14y^4+149y^3-153y^2-13y+31-(18y^3+138y^2-84y)\ln y}{(y-1)^5}\Big], \tag{5.34}
$$

$$
\begin{aligned}
M_7^H(y) &= \frac{4y}{3}\Big[\frac{8y^2-28y+12}{3(y-1)^3}Li_2\big(1-\frac{1}{y}\big)+\frac{3y^2+14y-8}{3(y-1)^4}\ln^2 y \\
&\quad +\frac{4y^3-24y^2+2y+6}{3(y-1)^4}\ln y+\frac{-2y^2+13y-7}{(y-1)^3}\Big],
\end{aligned}
\tag{5.35}
$$

$$Z_7^H(y) \;=\; \frac{2y}{3}\Big[\frac{-8y^3+55y^2-68y+21-(6y^2+28y-16)\ln y}{(y-1)^4}\Big], \tag{5.36}$$

$$\begin{aligned} G_8^H(y) \;=\; & \frac{y}{6}\Big[\frac{13y^3-17y^2+30y}{(y-1)^4}Li_2\big(1-\frac{1}{y}\big)-\frac{17y^2+31y}{(y-1)^5}\ln^2 y \\ & +\frac{42y^4+318y^3+1353y^2+817y-226}{36(y-1)^5}\ln y \\ & +\frac{-4451y^3+7650y^2-18153y+1130}{216(y-1)^4}\Big]-\frac{1}{6}E_H, \end{aligned} \tag{5.37}$$

$$N_8^H(y)=\frac{y}{36}\Big[\frac{-7y^4+25y^3-279y^2+223y+38+(102y^2+186y)\ln y}{(y-1)^5}\Big] \tag{5.38}$$

$$\begin{aligned} M_8^H(y) \;=\; & \frac{y}{3}\Big[\frac{17y^2-25y+36}{2(y-1)^3}Li_2\big(1-\frac{1}{y}\big)-\frac{17y+19}{(y-1)^4}\ln^2 y \\ & +\frac{14y^3-12y^2+187y+3}{4(y-1)^4}\ln y-\frac{3(29y^2-44y+143)}{8(y-1)^3}\Big], \end{aligned} \tag{5.39}$$

$$Z_8^H(y) \;=\; \frac{y}{6}\Big[\frac{-7y^3+23y^2-97y+81+(34y+38)\ln y}{(y-1)^4}\Big], \tag{5.40}$$

The NLO contributions to the Wilson coefficients at the scale $\mu = m_b = \mu_b$ are given by [113]

$$\begin{aligned} C_7^{(1)\mathrm{eff}}(\mu_b) \;=\; & \eta^{\frac{39}{23}}C_7^{(1)\mathrm{eff}}(\mu_W)+\frac{8}{3}(\eta^{\frac{37}{23}}-\eta^{\frac{39}{23}})C_8^{(1)\mathrm{eff}}(\mu_W)+\frac{37208}{4761}(\eta^{\frac{39}{23}}-\eta^{\frac{16}{23}})C_7^{(0)\mathrm{eff}}(\mu_W) \\ & +\big(\frac{297664}{14283}\eta^{\frac{16}{23}}-\frac{7164416}{357075}\eta^{\frac{14}{23}}+\frac{256868}{14283}\eta^{\frac{37}{23}}-\frac{6698884}{357075}\eta^{\frac{39}{23}}\big)C_8^{(0)\mathrm{eff}}(\mu_W) \\ & +\sum_{i=1}^{8}\Big[e_i\eta C_4^{(1)\mathrm{eff}}(\mu_W)+(f_i+k_i\eta)C_2^{(0)\mathrm{eff}}(\mu_W)+l_i\eta C_1^{(1)\mathrm{eff}}(\mu_W)\Big]\eta^{a_i}, \end{aligned} \tag{5.41}$$

where the vectors e_i, f_i, k_i, and l_i are given in the Appendix C of Ref. [113].

The contribution of the charged Higgs particle to the Wilson coefficients at the scale μ_b can be written as [113]

$$C_i^{\mathrm{eff}}(\mu_b)=C_{i\mathrm{SM}}^{\mathrm{eff}}(\mu_b)+\cot^2\Big(G_7^H(y)+N_7^H(y)\ln\frac{\mu_W^2}{m_W^2}\Big)+\Big(M_7^H(y)+Z_7^H(y)\Big) \tag{5.42}$$

The bremsstrahlung contributions should be taken into account in the NLO case where the corresponding decay width $\Gamma(\bar{b}\to s\gamma g)$ takes the form

$$\Gamma(\bar{b}\to s\gamma g)=\frac{G_F^2}{32\pi^4}|V_{ts}^*V_{tb}|^2\alpha_{\mathrm{em}}m_b^5 A \tag{5.43}$$

where [115]

$$A = (e^{-\alpha_s(\mu)\ln(\delta)(7+2\ln(\delta))/(3\pi)} - 1) + \frac{\alpha_s(\mu)}{\pi} \sum_{i,j;i\le j}^{8} \mathrm{Re}\,(C_i^{\mathrm{eff}}(\mu) C_j^{\mathrm{eff}}(\mu) f_{(ij)}(\delta) \tag{5.44}$$

where f_{ij} is defined as [106]

$$\begin{aligned}
f_{11}(\delta) &= \frac{1}{36} f_{22}(\delta), \quad f_{12}(\delta) = -\frac{1}{3} f_{22}(\delta), \quad f_{17}(\delta) = -\frac{1}{6} f_{27}(\delta), \quad f_{18}(\delta) = -\frac{1}{6} f_{28}(\delta), \\
f_{22}(\delta) &= \frac{16z}{27}\Big[\delta \int_0^{(1-\delta)/z} dt(1-zt)\Big|\frac{G(t)}{t} + \frac{1}{2}\Big|^2 + \int_{(1-\delta)/z}^{1/z} dt(1-zt)^2\Big|\frac{G(t)}{t} + \frac{1}{2}\Big|^2\Big], \\
f_{27}(\delta) &= -\frac{8z^2}{9}\Big[\delta \int_0^{(1-\delta)/z} dt \mathrm{Re}\,(G(t) + \frac{t}{2}) + \int_{(1-\delta)/z}^{1/z} dt(1-zt)\mathrm{Re}\,\big(G(t) + \frac{t}{2}\big)\Big], \\
f_{28}(\delta) &= -\frac{1}{3} f_{27}(\delta)
\end{aligned} \tag{5.45}$$

The branching ratio of the inclusive radiative decay $\bar{B} \to X_s\gamma$ at the NLO can be found in [106, 115]

$$\mathcal{B}(\bar{B} \to X_s\gamma) = \mathcal{B}(\bar{B} \to X_c e\bar{\nu}_e)_{\mathrm{exp}} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_e}{\pi f(z) k(z)} \frac{m(\mu_b)}{m_b^2} (|D|^2 + A)\Big(1 - \frac{\delta_{\mathrm{SL}}^{\mathrm{NP}}}{m_b^2} + \frac{\delta_\gamma^{\mathrm{NP}}}{m_b^2} + \frac{\delta_c^{\mathrm{NP}}}{m_c^2}\Big) \tag{5.46}$$

where the QCD corrections [115, 127] for the semileptonic decay are given by

$$k(z) = 1 - \frac{2\alpha_s(\mu_b)}{3\pi} \frac{h(z)}{f(z)} \tag{5.47}$$

with

$$\begin{aligned}
h(z) &= -(1-z^2)\big(\frac{25}{4} - \frac{239}{3}z + \frac{25}{4}z^2\big) + z\ln z\big(20 + 90z - \frac{4}{3}z^2 + \frac{17}{3}z^3\big) \\
&\quad + z^2\ln^2 z(36 + z^2) + (1-z^2)\big(\frac{17}{3} - \frac{64}{3}z + \frac{17}{3}z^2\big) \\
&\quad - 4(1 + 30z^2 + z^4)\ln z \ln(1-z) - (1 + 16z^2 + z^4)(6Li_2(z) - \pi^2) \\
&\quad - 32z^{3/2}(1+z)\Big(\pi^2 - 4Li_2(\sqrt{z}) + 4Li_2(-\sqrt{z}) - 2\ln z \ln\big(\frac{1-\sqrt{z}}{1+\sqrt{z}}\big)\Big)
\end{aligned} \tag{5.48}$$

The heavy-quark effective theory is used to relate the quark decay rate to the actual hadronic process. In terms of the contributions to heavy hadron mass from the kinetic energy (λ_1) and chromo-magnetic interactions (λ_2), the $1/m_b^2$ corrections to the semileptonic and radiative decays appearing in Eq. (5.46) are given by

$$\begin{aligned}
\delta_{\mathrm{SL}}^{\mathrm{NL}} &= \frac{\lambda_1}{2} + \frac{3}{2}\lambda_2\Big(1 - 4\frac{(1-z)^4}{f(z)}\Big) \\
\delta_\gamma^{\mathrm{NL}} &= \frac{\lambda_1}{2} - \frac{9}{2}\lambda_2
\end{aligned} \tag{5.49}$$

and the non-perturbative corrections to the semileptonic decay that comes from $1/m_c^2$ takes the form

$$\delta_c^{\rm NL} = -\frac{1}{9}\frac{\lambda_2}{C_7^{(0)\rm eff}(\mu_b)}\Big(C_2^{(0)\rm eff}(\mu_b) - \frac{C_1^{(0)\rm eff}(\mu_b)}{6}\Big) \tag{5.50}$$

The NLO term corresponding to the Wilson coefficient $C_7^{(0)\rm eff}$ at the LO is given by

$$D = C_7^{(0)\rm eff}(\mu_b) + \frac{\alpha_s}{4\pi}\Big(C_7^{(1)\rm eff}(\mu_b) + \sum_{i=1}^{8} C_7^{(0)\rm eff}(\mu_b)\big[r_i + \gamma_{i7}^{(0)\rm eff}\ln\frac{m_b}{\mu_b}\big]\Big) \tag{5.51}$$

where the coefficients r_i at the NLO have been calculated in Ref. [122] and $\gamma_{i7}^{(0)\rm eff}$ can be found in Ref. [106].

The current SM prediction is based on NLO and dominant NNLO contributions (for recent references, see [128]). Since the corresponding level of precision is not available for the 2HDM, we shall take as *reference* value the NLO SM prediction, rather than the most precise theoretical value or the experimental value. As input parameters for (5.54) we take the CKM ratio to be 0.967 [129], $\mathcal{B}(B \to X_c e\bar{\nu}_e) = 0.1095$ [130], $m_t = 163$ GeV (corresponding to a pole mass of $m_t = 172.7$ GeV), $m_c = 1.224$ GeV [129] and $\mu_b = m_b = 4.68$ GeV [129], yielding $m_c/m_b = 0.262$.

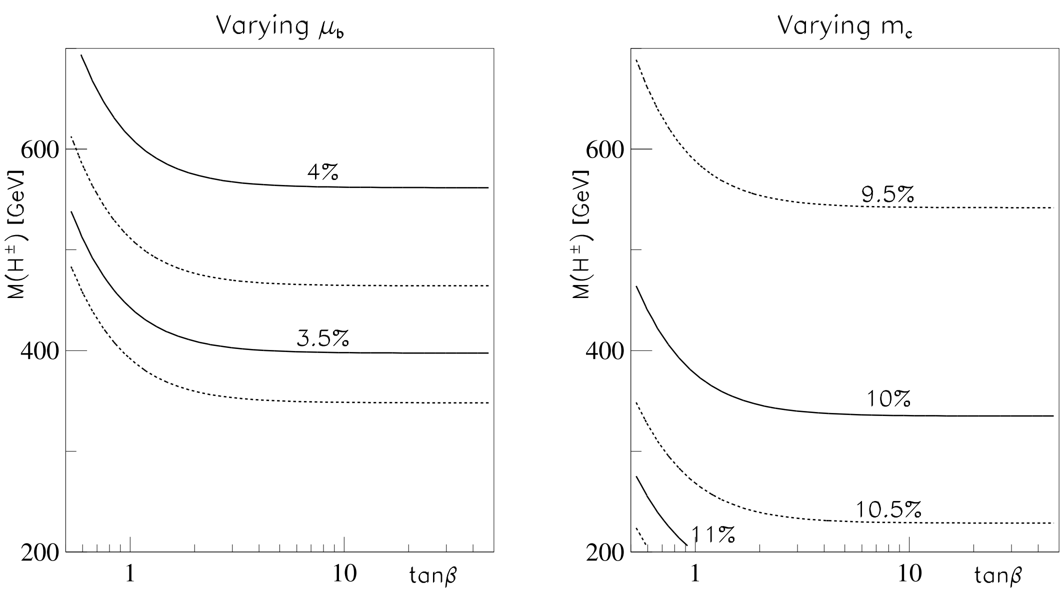

Figure 5.1: Contributions to the uncertainty in the decay rate due to mass scale uncertainties. Left panel: due to uncertainty in the μ_b scale; and right panel: due to uncertainty in m_c; both vs. $\tan\beta$ and $M_{H^\pm}$.

For the uncertainty that enters in Eq. (5.46), we take

$$\sigma[\mathcal{B}(B \to X_s\gamma)] = 15\% \times \mathcal{B}(B \to X_s\gamma)_{\rm ref}. \tag{5.52}$$

This value, which is a somewhat conservative estimate, should be compared with the current experimental precision (see above) and the currently best SM theoretical prediction, which has an uncertainty of about 10% [128]. The latter is dominated by scale (μ_b, μ_c, μ_W) ambiguities and parameter uncertainties. For the 2HDM, the theoretical studies have not been carried to the same level of precision, and also, at the NLO level, the uncertainties are known to be larger than in the SM [109,113]. Furthermore, they depend on the 2HDM parameters.

Fig. 5.1 is devoted to a study of how the uncertainties depend on $\tan\beta$ and $M_{H^\pm}$. The uncertainties in the 2HDM predictions are normalized with respect to the SM prediction, i.e., we show $\delta(\mathcal{B}_{\rm 2HDM})/\mathcal{B}_{\rm SM}$. In the left panel, we display the maximum variation in the brancing ratio, obtained from varying the scale μ_b between $m_b/2$ and $2m_b$. These values, of around 3–4%, may be compared with 3.2% found for the SM [123].

In the right panel, we similarly show how the branching ratio changes when we reduce m_c by a factor 0.758 (to 0.928 GeV) from the default value 1.224 GeV [129]. This reduction corresponds to changing from the pole mass to the $\overline{MS}$ current mass [124], and yields variations from 9.5% to 11%. Alternatively, following [131], with 1.15 GeV $\leq m_c \leq$ 1.45 GeV, we find an uncertainty ranging from 10% to over 12% (at low $M_{H^\pm}$ and low $\tan\beta$).

A more detailed estimate of the dependence of the decay rate on the m_c scale is available for the SM, taking into account dominant higher-order effects [126], and gives an uncertainty of about 9.5%. (The earlier study of Gambino and Misiak [124] gave an uncertainty of 11%.) For the 2HDM no correspondingly detailed analysis is available.

Two further comments are in order, regarding the combination of uncertainties:

(a) The uncertainty to μ_b increases somewhat with $M_{H^\pm}$, whereas that related to m_c decreases. Together, these dependences on $M_{H^\pm}$ partly cancel.

(b) The theory uncertainties are not gaussian, and therefore can not be added in quadrature to the experimental ones.

In the next subsection we will see the Next-to-Next-to-Leading Order (NNLO) contribution to the SM, and see the effects of these results on the 2HDM.

5.1.3 NNLO contributions

The main theoretical uncertainty in the SM prediction for $\mathcal{B}(\bar{B}\to X_s\gamma)$ originates from the perturbative calculation of the $b\to s\gamma$ amplitude. The calculation of the three loop diagrams at the NNLO QCD corrections removes the charm-quark mass renormalization ambiguity which comes from the calculation of the two loop diagrams at the NLO QCD corrections. The NNLO matching was calculated in Refs. [132, 133] and three-loop renormalization for the operators $Q_1,\ldots,Q_8$ was found in Refs. [134, 135].

The μ_b dependence of the NNLO branching ratio is very weak ($\sim$ 1.4%) and the dependence on the μ_W is not strong, so according to Gambino et al [124], the NNLO is simply of order $\alpha_s(\mu_b)\approx 0.5\%$ up to an unkown factor of order unity.

The effective Wilson coefficients at NNLO can be expanded in powers of α_s as follows

$$C_i^{\rm eff}(\mu) = C_i^{(0)\rm eff} + \frac{\alpha_s}{4\pi}C_i^{(1)\rm eff} + \big(\frac{\alpha_s}{4\pi}\big)^2 C_i^{(2)\rm eff} + \ldots \tag{5.53}$$

At the NNLO, the branching ratio can be written as [124, 136]:

$$\mathcal{B}(\bar{B} \to X_s \gamma) = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{\text{e.m.}}}{\pi C} \{P(E_0) + N(E_0)\} \times \mathcal{B}(\bar{B} \to X_c e \bar{\nu}_e)_{\text{exp}}, \tag{5.54}$$

where P and N denote perturbative and non-perturbative effects that both depend on the photon lower cut-off energy E_0. In the LO limit, the term P reduces to the square of the effective Wilson coefficient $C_7^{(0)\text{eff}}(\mu_b)$ in (5.9).

The SM prediction of Misiak *et al.* is $(3.15 \pm 0.23) \times 10^{-4}$ for $E_0 = 1.6$ GeV [136, 137], if all errors are added in quadrature. This is to be compared with the recent experimental results, which are averaged to 3.55×10^{-4} [138], with an uncertainty of 7–7.5%, again with statistical and systematic errors added in quadrature.

The 2HDM contribution is positive, a finite value for the charged Higgs mass would thus bring the results of [137, 139] in closer agreement with the experiment. We shall however take the attitude that these numbers are compatible and compare the uncertainty in the experimental result and the SM prediction with the 2HDM contribution.

In the NNLO, the perturbative contribution P is obtained via the following three steps [136]:

1. Evaluation of the Wilson coefficients at the "high" (electroweak) scale, μ_0 [132, 133]. These coefficients are expanded to second order in α_s and rotated to "effective" Wilson coefficients $C_i^{\text{eff}}(\mu_0)$ [109, 140]. The 2HDM effects enter at this stage, at lowest order in the Wilson coefficients $C_7(\mu_0)$ and $C_8(\mu_0)$.

2. Evaluation of the "running" and mixing of these operators, from the high scale to the "low" (B-meson) scale. This is where the main QCD effects enter via a matrix U that is given in terms of powers of $\eta = \alpha_s(\mu_0)/\alpha_s(\mu_b)$ [106, 109, 135, 140].

3. Evaluation of matrix elements at the low scale [136, 141], which amounts to constructing $P(E_0)$ of Eq. (5.54) from the $C_i^{\text{eff}}(\mu_b)$.

The NNLO improve the prediction of the branching ratio and reduce the uncertainty dependence on the scale. For more details see Ref. [19]. In the next section we will discuss the decay rate $B \to \tau \nu_\tau$ and see how it can constrain the parameter space of the 2HDM.

5.2 The constraint of the $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)$ decay

The purely leptonic decay $B^- \to \tau^- \bar{\nu}_\tau$ in the SM proceeds via annihilation of b and $\bar{u}$ quarks to a W^- and this decay also is valid for the charge conjugate states. It provides a direct determination of the product of the B meson decay constant f_B and the magnitude of the CKM matrix element $|V_{ub}|$. The branching ratio is given by [142]

$$\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \Big(1 - \frac{m_\tau^2}{m_B^2}\Big)^2 f_B^2 |V_{ub}|^2 \tau_B \tag{5.55}$$

where m_b and m_τ are the B and τ masses respectively, and τ_b is the B^- lifetime.

The charged Higgs boson modifies the SM branching ratio by the factor

$$r_H = \left[1 - \frac{m_B^2}{m_{H^\pm}^2} \tan^2\beta\right]^2 \tag{5.56}$$

The branching ratio for the 2HDM then takes the form [142]

$$\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B r_H \tag{5.57}$$

The recent measurement of $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)$ is $(1.79 \pm 0.71) \times 10^{-4}$ [143] with the SM prediction of the branching ratio $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = (1.59 \pm 0.40) \times 10^{-4}$, one can extract the value of $r_H = 1.13 \pm 0.53$ which puts a new bound

$$\tan\beta < 0.09\ m_{H^\pm}\ (\text{GeV})^{-1} \tag{5.58}$$

where the old bound is [142]

$$\tan\beta < 0.52\ m_{H^\pm}\ (\text{GeV})^{-1} \tag{5.59}$$

This bound excludes the lower right corner of the Fig(2) in Ref. [19].

In the next section we will study the $B - \bar{B}$ oscillation.

5.3 $B - \bar{B}$ oscillation constraint

Oscillations between particle and antiparticle were predicted for the $K^0 - \bar{K}^0$ system in 1955 [144] and observed in 1956 [145]. This oscillation was observed later in the neutral $B^0 - \bar{B}^0$ meson system [146,147]. The flavour eigenstates of the B^0 meson can be written as

$$B_d^0 = (\bar{b}d), \quad \bar{B}_d^0 = (b\bar{d}), \quad B_s^0 = (\bar{b}s), \quad \bar{B}_s^0 = (b\bar{s}) \tag{5.60}$$

The heavy and light mass eigenstates take the form

$$B_H = pB^0 + q\bar{B}^0, \quad B_L = pB^0 - q\bar{B}^0, \tag{5.61}$$

with

$$p = \frac{1 + \epsilon_b}{\sqrt{2(1 + |\epsilon_B|^2)}}, \quad q = \frac{1 - \epsilon_b}{\sqrt{2(1 + |\epsilon_B|^2)}} \tag{5.62}$$

where ϵ_B is small complex parameter (responsible for CP violation). The oscillation of $B^0 - \bar{B}^0$ is given by the phenomenological Hamiltonian matrix H [148]

$$H \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} \tag{5.63}$$

The diagonal terms describe the decay of the neutral B mesons, with M being the mass of the flavour eigenstates B^0 and $\bar{B}^0$, and Γ their decay width. The off-diagonal terms are responsible for the $B^0-\bar{B}^0$ transitions through second order weak interaction. M_{12} corresponds to virtual $B^0-\bar{B}^0$ transitions, while Γ_{12} describes real transitions due to decay modes. The mass difference between mass eigenstates is a measure of the oscillation frequency of change from B^0 to $\bar{B}^0$ and vice versa and for the SM it is given by [149]

$$\Delta m_q = \frac{1}{2m_{B_q}}\langle B_q^0|H_{\text{eff}}|\bar{B}_q^0\rangle = M_H^q - M_L^q = 2M_{12} \tag{5.64}$$

$$= \frac{G_F^2}{6\pi^2}|V_{tq}^* V_{tb}|^2 m_W^2 \eta_{B_q} f_{B_q}^2 B_{B_q} S_0(x_t) m_{B_q}, \quad q=d,s \tag{5.65}$$

where f_{B_q} is the weak B decay constant, B_{B_q} the bag parameter of the B meson and the Inami-Lim function is defined as [107, 150]

$$S_0(x) = \frac{4x-11x^2+x^3}{4(1-x)^2} - \frac{3x^3}{2(1-x)^3}\ln x, \tag{5.66}$$

and η_{B_q} stands for short distance QCD corrections.

In the 2HDM, there is a new contribution to $B^0-\bar{B}^0$ oscillation coming from the charged Higgs boson $H^\pm$. Therefore the measurement of the mass splitting can be used to constrain the new physics i.e, 2HDM. The mass difference for $B^0-\bar{B}^0$ oscillation at the LO takes the form [152]

$$\Delta m_{B_q} = \frac{G_{\text{F}}^2}{6\pi^2}|V_{tq}^* V_{tb}|^2 m_W^2 \eta_{B_q} f_{B_q}^2 B_{B_q} S_{2HDM} m_{B_q}, \quad q=d,s \tag{5.67}$$

with the Inami–Lim functions [107]

$$S_{2HDM_{LO}} = S_{WW_{LO}} + S_{WH_{LO}} + S_{HH_{LO}}. \tag{5.68}$$

with

$$\begin{aligned} S_{WW_{LO}} &= \frac{x}{4}\Big[1+\frac{3-9x}{(x-1)^2}+\frac{6x^2\ln x}{(x-1)^3}\Big], \\ S_{WH_{LO}} &= xy\cot^2\beta\Big[\frac{(4z-1)\ln y}{2(1-y)^2(1-z)} - \frac{3\ln x}{2(1-x)^2(1-z)} + \frac{x-4}{2(1-x)(1-y)}\Big], \\ S_{HH_{LO}} &= \frac{xy\cot^2\beta}{4}\Big[\frac{1+y}{(1-y)^2}+\frac{2y\ln y}{(1-y)^3}\Big] \end{aligned} \tag{5.69}$$

where $x=m_t^2/m_W^2$, $y=m_t^2/m_H^2$, and $z=m_W^2/m_H^2$.

The NLO corrections enhance the LO results for Δm_q by 18% [151].

$$\Delta m_{B_q} = \frac{G_{\text{F}}^2}{6\pi^2}|V_{tq}^* V_{tb}|^2 m_W^2 \eta_2(x_W, x_H) f_{B_q}^2 B_{B_q} S_{2HDM} m_{B_q}, \quad q=d,s \tag{5.70}$$

where $x_W = m_t^2/m_W^2$ and $x_H = m_t^2/m_H^2$, with the Inami–Lim functions [107]

$$S_{2HDM_{NLO}} = S_{WW_{NLO}} + 2S_{WH_{NLO}} + S_{HH_{NLO}}. \tag{5.71}$$

with

$$B_{B_q =} B_{B_q}(\mu)\alpha_s(\mu)^{-6/23}\Big[1-\frac{\alpha_s(\mu)}{4\pi}\Big(\frac{\gamma_1^m}{2\beta_0}-\frac{\gamma_0^m}{2\beta_0^2}\beta_1\Big)\Big] \tag{5.72}$$

and

$$\eta_2(x_W, x_H) = \alpha_s(m_W)^{\gamma_0^m/(2\beta_0)} \Big[1 + \frac{\alpha_s(m_W)}{4\pi}\Big(\frac{D_{2HDM_{NLO}}(x_W, x_H)}{S_{2HDM_{NLO}}(x_W, x_H)} + Z\Big)\Big] \tag{5.73}$$

with

$$Z = \frac{\gamma_1^m}{2\beta_0} - \frac{\gamma_0^m}{2\beta_0^2}\beta_1 \tag{5.74}$$

where $D_{2HDM_{NLO}}(x_W, x_H)$ and $S_{2HDM_{NLO}}(x_W, x_H)$ are defined in Ref. [151].

A few comments are here in order: (i) The factor of 2 for the second term in (5.71) has been adopted to follow the notation of [151]. (ii) There is a book-keeping problem with the expression for $L^{(i,H)}$ in Eq. (A.20) of [151]. Since the last term in that expression, proportional to a quantity denoted HH, has an explicit coefficient $1/\tan^4\beta$, the S_{HH} in Eqs. (A.21) and (A.24) for $HH^{(i)}$ should be replaced by $\tan^4\beta \times S_{HH}$. (iii) There is a discrepancy in a quantity denoted $2WW_{tu}^{(8)}$, between Eq. (A.16) in [151] and the later PhD thesis of the same author. We have chosen to take the formula given in the thesis. At the level of η_2, it amounts to a difference of the order of 2. The $\mathcal{O}(\alpha_s)$ corrections to η introduce a variation of η (or η_2) from 0.334 at $\tan\beta = 0.5$ and $M_{H^\pm} = 200$ GeV to 0.552 at $\tan\beta = 50$. (We have adopted the over-all normalization of η such that it agrees with the SM value 0.552 [153] at $\tan\beta = 50$.) However, the product $\eta \times S_{2HDM}$ varies by a factor of 2.7 over this same range, as compared with a factor of 4.5 for S_{2HDM} itself. Thus, the inclusion of the $\mathcal{O}(\alpha_s)$ QCD corrections reduce the sensitivity of Δm_B to charged-Higgs contributions. In other words, the inclusion of NLO corrections weakens the constraints on the 2HDM at low values of $\tan\beta$.

In the next chapter we will study the experimental constraints on the neutral sector of the 2HDM.

Chapter 6

Experimental constraints on the neutral-Higgs sector

This chapter is an extension to the work presented in the following publishing papers

- Paper 1 "Consistency of the two Higgs doublet model and CP violation in top production at the LHC", Nucl. Phys. B775:45-77, 2007.

- Paper 2 "Constraining the Two-Higgs-Doublet-Model parameter space", Phys. Rev. D76:095001, 2007.

This work also represents the calculations of the experimental constraints on the neutral sector of the 2HDM such as the precise measurements of R_b, non-observation of a neutral Higgs boson at LEP2, $\Delta\rho$, and $a_\mu = \frac{1}{2}(g-2)_\mu$ and the combinations of these constraints. We will study the effects of changing the neutral Higgs mass M_2 and μ^2 to each constraint and the combination of them by using the neutral Higgs mass $M_1 =$ 150 GeV (greater than the lower limit for the neutral Higgs of the SM $M \sim 114$ GeV), 80 GeV, and 114 GeV. This study shows how the allowed region of the parameter space in the 2HDM changes with M_2 and μ^2. We will demonstrate the results of each constraint in the following sections.

In the next section we will study the partial decay width R_b for the process $Z \to b\bar{b}$.

6.1 R_b constraint

In the SM, the $Zb\bar{b}$ couplings receive a correction from the exchange of the longitudinal components of the $W^\pm$ and Z bosons. In the 't Hooft gauge the longitudinal components of the $W^\pm$ and Z are replaced by the Goldstone bosons $G^\pm$ and G^0. In this gauge the Goldstone bosons are physical degrees of freedom and have masses m_W and m_Z respectively.

In the SM, virtual Higgs exchange does contribute to the decay $Z \to b\bar{b}$, the coupling of the neutral Higgs h to b quarks is too small to make an observable contribution but the coupling of the charged Goldstone $G^\pm$ to $t\bar{b}$ is large enough to make an observable contribution to $Z \to b\bar{b}$ [170]. The tree-level $Zb\bar{b}$ couplings in the SM are given by

$$g^L_{Zb\bar{b}} = \frac{e}{s_W c_W}\Big(-\frac{1}{2}+\frac{1}{3}s_W^2\Big) \tag{6.1}$$

$$g^R_{Zb\bar{b}} = \frac{e}{s_W c_W}\Big(\frac{1}{3}s_W^2\Big) \tag{6.2}$$

where $g^{L,R}_{Zb\bar{b}}$ are the left- and right-handed $Zb\bar{b}$ couplings. The radiative corrections to $Z \to b\bar{b}$ change the $Zb\bar{b}$ couplings from their tree-level values by $\delta g^{L,R}$ (correction of the new physics to the left- and right-handed $Zb\bar{b}$ couplings, respectively)

$$\bar{g}^{L,R}_{Zb\bar{b}} = g^{L,R}_{Zb\bar{b}} + \delta g^{L,R} \tag{6.3}$$

The hadronic branching ratio of Z to b quarks R_b is given by

$$R_b \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} = \frac{\Gamma_b}{\Gamma_h} = \frac{\Gamma_b}{3\Gamma_b + 2\Gamma_c} \tag{6.4}$$

where Γ_c is the width of the process $Z \to c\bar{c}$.

The electroweak oblique and QCD corrections to the $Zb\bar{b}$ branching ratio R_b cancel between numerator and denominator, R_b is very sensitive to the new physics beyond the SM, so that the precision experimental value of R_b gives a severe constraint on the new physics [171]. The R_b measurement can be used to set lower bounds on the charged Higgs mass. The 2HDM contributes corrections to the process $Z \to b\bar{b}$ through the charged Higgs couplings to $t\bar{b}$ and the neutral Higgs couplings to $b\bar{b}$ [170]. In the 2HDM with CP violation, the relevant couplings for the neutral Higgs bosons which contribute to the $Zb\bar{b}$ vertex are the ZH_iH_j, ZH_iG^0 with all incoming momenta [17]

$$\begin{aligned}
ZH^+H^-: &\quad \frac{-ig\cos 2\theta_W}{2\cos\theta_W}(p^+_\mu - p^-_\mu),\\
ZG^+G^-: &\quad \frac{-ig\cos 2\theta_W}{2\cos\theta_W}(p^+_\mu - p^-_\mu),\\
ZH_jH_k: &\quad \frac{-g}{2\cos\theta_W}[(\sin\beta R_{j1} - \cos\beta R_{j2})R_{k3} - (\sin\beta R_{k1} - \cos\beta R_{k2})R_{j3}](p^j_\mu - p^k_\mu),\\
ZH_jG^0: &\quad \frac{g}{2\cos\theta_W}(\cos\beta R_{j1} + \sin\beta R_{j2})(p^j_\mu - p^0_\mu),
\end{aligned} \tag{6.5}$$

The contributions from 2HDM take the form [170]

$$\delta g^L = \frac{1}{32\pi^2}\Big(\frac{gm_t}{\sqrt{2}m_W}\cot\beta\Big)^2\frac{e}{s_W c_W}\Big[\frac{R}{R-1} - \frac{R\ln R}{(R-1)^2}\Big] \tag{6.6}$$

$$\delta g^R = \frac{1}{32\pi^2}\Big(\frac{gm_b}{\sqrt{2}m_W}\tan\beta\Big)^2\frac{e}{s_W c_W}\Big[\frac{R}{R-1} - \frac{R\ln R}{(R-1)^2}\Big] \tag{6.7}$$

where $R = m_t^2/m_{H^+}^2$, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ and θ_W is the Weinberg angle. This correction in addition to the corrections due to Goldstone boson exchange (the SM contributions) give the contributions of the 2HDM to the process $Z \to b\bar{b}$.

For small $\tan\beta$ the neutral Higgs couplings to b quarks are small, and the contributions to $Z \to b\bar{b}$ due to neutral Higgs exchange can be neglected. In this regime the corrections due to charged Higgs exchange can be used to constrain the 2HDM. At large $\tan\beta$ the neutral Higgs couplings to b quarks are large and the contributions to $Z \to b\bar{b}$ due to neutral Higgs exchange are significant, but the large values of $\tan\beta$ are excluded by the unitarity constraint [19].

For the CP-conserving case, the 2HDM contributions to R_b were given in [164]. In the general CP-violating case, the charged-Higgs contribution, Eq. (4.2) of [164], remains unchanged, but we find that the neutral-Higgs part, Eq. (4.4), gets modified to

$$\begin{aligned}
\delta\Gamma_V^f(H) = \frac{\alpha_{\rm e.m}^2 N_c\, m_Z}{96\pi \sin^4\theta_W \cos^2\theta_W}\, \frac{m_b^2}{m_W^2} \Bigg\{ \sum_{j=1}^{3} \Bigg[\sum_{k=1}^{3} [(\sin\beta R_{j1} - \cos\beta R_{j2}) R_{k3} \\
- (\sin\beta R_{k1} - \cos\beta R_{k2}) R_{j3}] \frac{\tan\beta}{\cos\beta}\, R_{j1} R_{k3}\, \rho_4(m_Z^2, M_j^2, M_k^2, 0) \\
- (\cos\beta R_{j1} + \sin\beta R_{j2}) \frac{R_{j1}}{\cos\beta}\, \rho_4(m_Z^2, M_j^2, m_Z^2, 0) \\
+ 2Q_b \sin^2\theta_W (1 + 2Q_b \sin^2\theta_W) \left(\frac{R_{j1}^2}{\cos^2\beta} + \tan^2\beta R_{j3}^2 \right) \rho_3(m_Z^2, M_j^2, 0) \Bigg] \\
+ 2Q_b \sin^2\theta_W (1 + 2Q_b \sin^2\theta_W)\, \rho_3(m_Z^2, m_Z^2, 0) \Bigg\}. \qquad (6.8)
\end{aligned}$$

The functions ρ_3 and ρ_4 are various combinations of three-point and two-point loop integrals [164]. For the numerical studies, we use the `LoopTools` package [175, 176]. It was found that R_b excludes the very small $\tan\beta$ and small charged Higgs mass $m_{H^\pm}$ and large $\tan\beta$.

In Paper 2 [19], we show the implication of the R_b constraint for the case when M_3 is calculated from the other input, i.e., M_3 is calculated from the relation (4.45). Here and in the next sections, we are going to study the case of a fixed value of M_3. Thus, we here follow the approach of Paper 1 [17] and therefore, we can not directly compare results here with those in [19].

In this study, calculations for the R_b constraint are different from the calculations made in our paper of Ref. [19] as follows:

- We take M_3 fixed at 500 GeV, whereas in Ref. [19] we calculate M_3 from the relation (4.45).
- Here we use $M_1 = 150$ GeV (greater than the lower limit of the SM Higgs $m_H \simeq 114$ GeV), in Ref. [19] we use $M_1 = 100$ GeV.
- We use in this study some new values of μ^2, such as $(400\text{ GeV})^2$, $-(300\text{ GeV})^2$, and $-(400\text{ GeV})^2$.

We will next show some figures for the R_b constraint, and select the following values for the parameters M_1, M_2, M_3 and μ^2 as:

- $M_1 = 150$ GeV.

- M_2 =(300, 500) GeV.
- $M_3 = 500$ GeV.
- $\mu^2 = (400\text{ GeV})^2, (300\text{ GeV})^2, 0, -(300\text{ GeV})^2, -(400\text{ GeV})^2$.

The 2HDM-specific contributions to R_b are of two kinds. At low values of $\tan\beta$, the exchange of charged Higgs bosons is important, whereas at high values of $\tan\beta$ the exchange of neutral-Higgs bosons is important [164].

The unitarity constraint excludes large values of $\tan\beta$ for different values of our parameters such as M_2 and μ^2. Thus, the neutral-Higgs exchange contribution to R_b is in this case of no importance. But the charged-Higgs exchange contribution to R_b remains important.

We notice that the allowed parameter space is reduced due to unitarity in the following cases: (i) $M_2^2 > \mu^2$ and (ii) $\mu^2 > M_2^2$, but the allowed regions of the parameter space in the 2HDM become large when $\mu^2 = M_2^2$ ($\mu = M_2 = 300$ GeV, see fig. 6.3) and there is more reduction as M_2 increases.

The R_b constraint excludes low values of $\tan\beta \lesssim 1$ for all cases considered (see figs. 6.1–6.10) and excludes large values of charged Higgs mass $m_{H^\pm} > 600$ GeV. The upper limit on $\tan\beta$ becomes smaller and the upper limit of the charged Higgs mass becomes larger as M_2 increases.

Figs. 6.1 and 6.2 are devoted to $\mu = 400$ GeV, with two values of M_2, 300 and 500 GeV.

The μ that is used in figs. 6.3 and 6.4 is 300 GeV. The upper limit of the charged Higgs mass $M_{H^\pm}$ becomes larger in the following cases (i) $M_1^2 < \mu^2 = M_2^2$ and (ii) $M_2^2 > \mu^2$ and excludes less parameter space compared to when $\mu^2 > M_2^2$ like in fig. 6.1.

In figs 6.5 and 6.6 we consider $\mu = 0$ GeV. The contribution of R_b in fig. 6.5 is the same as figs. 6.1–6.4 but in fig. 6.6 the upper limit on the charged Higgs mass $M_{H^\pm}$ is reduced because $\mu^2 = 0$ ($\mu^2 < M_1^2$).

Figs 6.7 and 6.8 are devoted to $\mu^2 = -(300\text{ GeV})^2$. The allowed region of the parameter space in the 2HDM is here dramatically reduced and the upper limit of $\tan\beta$ (the cutoff will be at $\tan\beta \sim 2$) and charged Higgs mass ($m_{H^\pm} \sim 500$ GeV) becomes smaller for large values of M_2. For negative values of μ^2 the unitarity constraint excludes large $\tan\beta$ and also low values of $\tan\beta$.

The μ^2 that is used in figs 6.9 and 6.10 is $-(400\text{ GeV})^2$, it leads to more exclusion of the parameter space than in figs. 6.7 and 6.8. The upper limit on $m_{H^\pm} \sim 300$ GeV becomes smaller for large values of M_2.

We conclude that for large values of M_1 and $M_2 = 500$ GeV more exclusion of the parameter space is done. The negative values of μ^2 exclude most of the parameter space due to unitarity and also exclude the low values of $\tan\beta$. The R_b constraint excludes the low values of $\tan\beta < 1$.

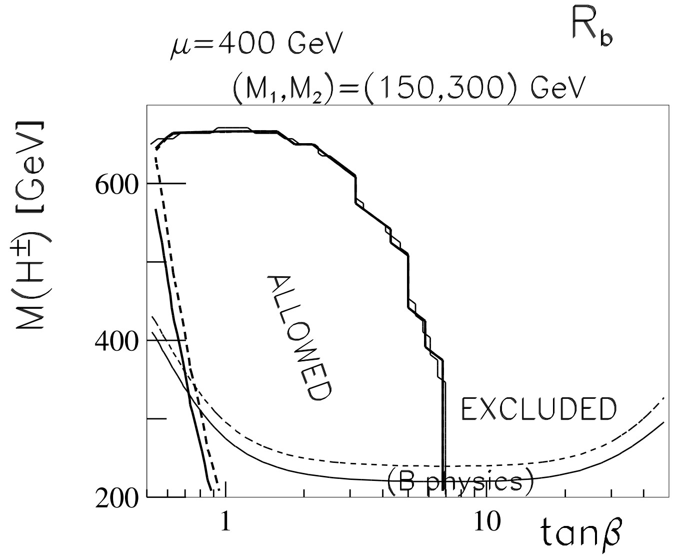

Figure 6.1: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

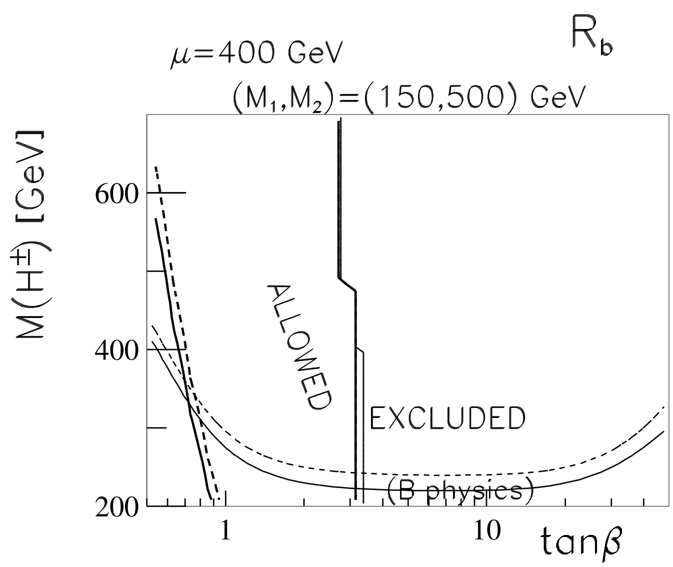

Figure 6.2: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

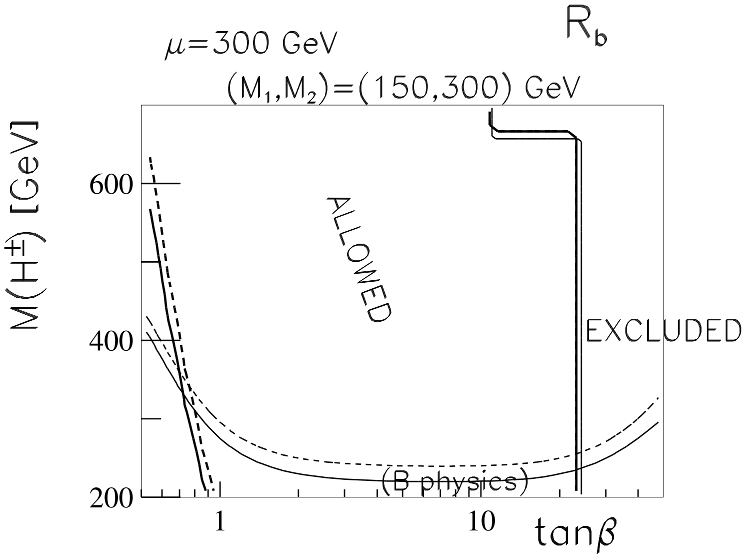

Figure 6.3: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

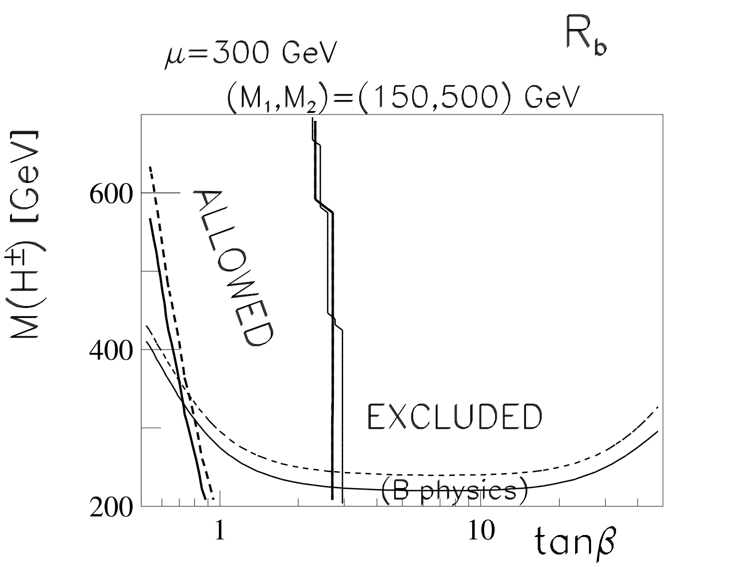

Figure 6.4: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

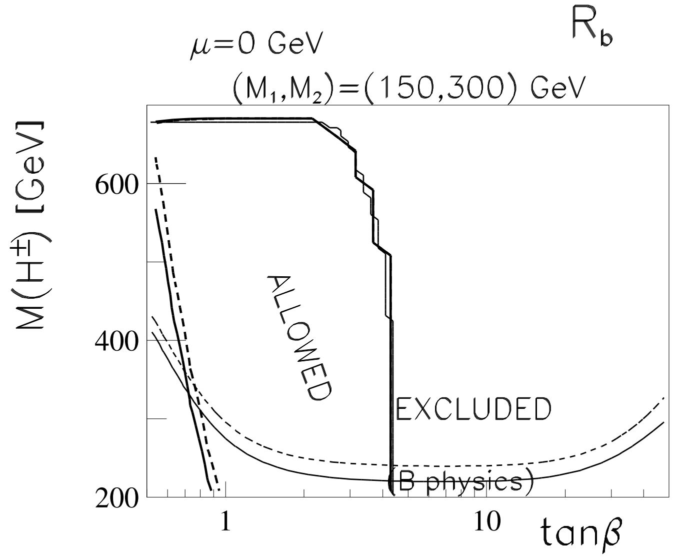

Figure 6.5: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

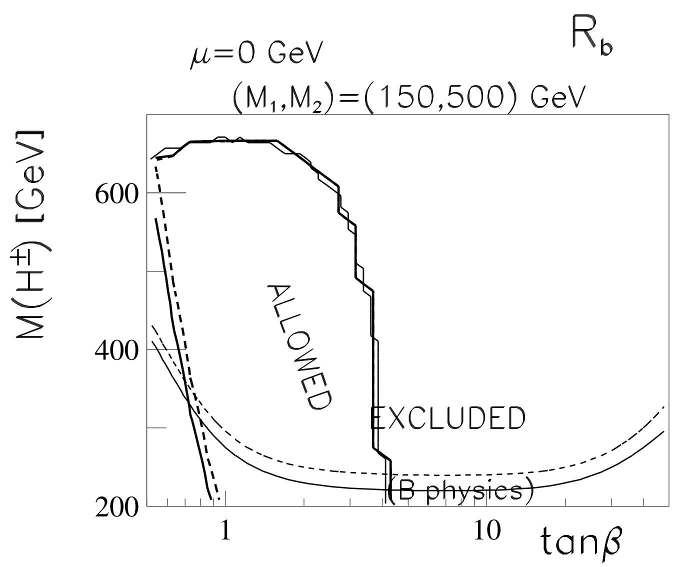

Figure 6.6: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

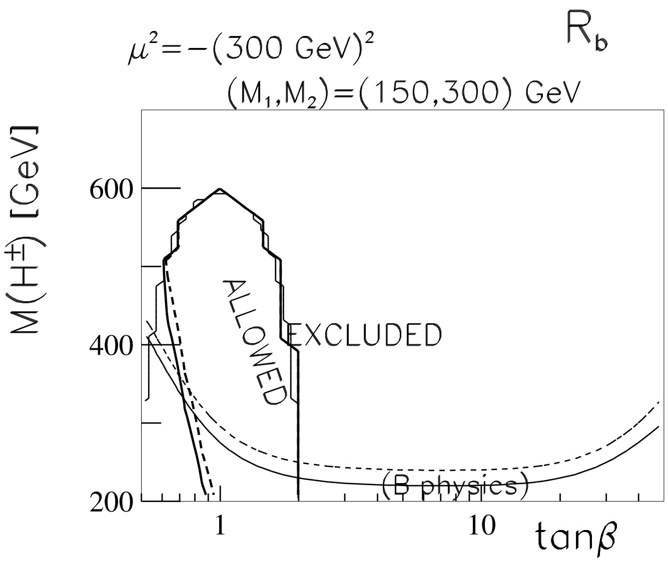

Figure 6.7: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

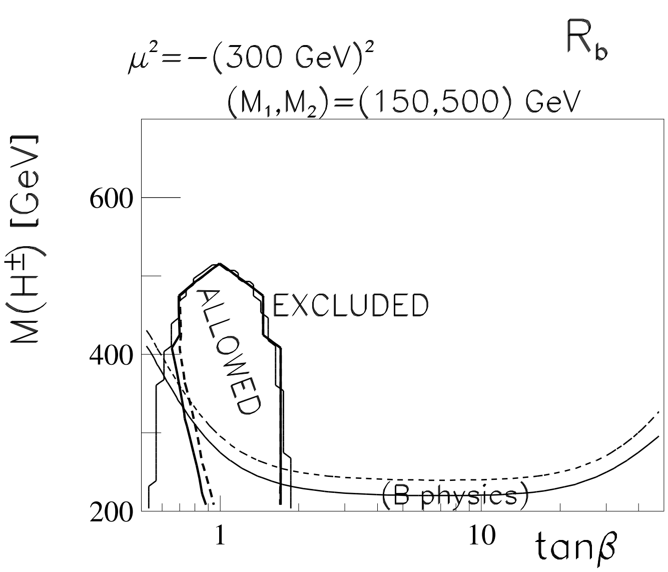

Figure 6.8: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

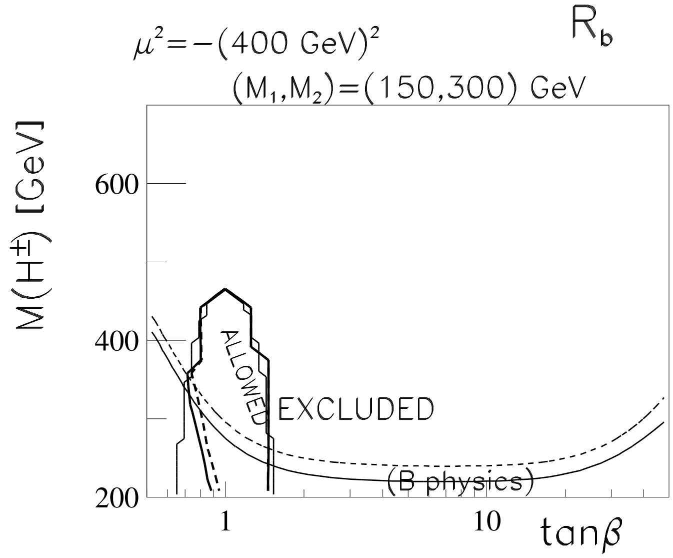

Figure 6.9: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

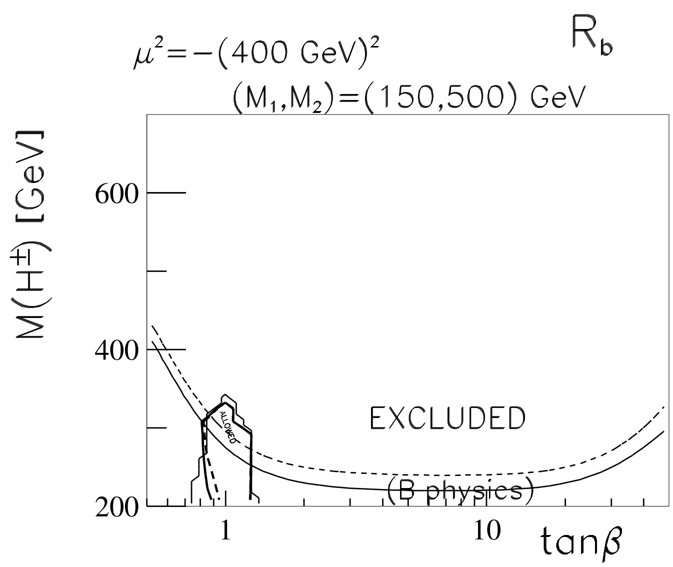

Figure 6.10: Excluded regions due to constraints from positivity, unitarity, R_b, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

In the next section we will study the experimental constraint a_μ.

6.2 The anomalous magnetic moment of the muon

The precise measurement of the muon anomaly a_μ or $(g-2)$ constitutes one of the more impressive tests not only for electroweak interactions but also of the strong interactions as well. The deviation of the SM from precision measurements Δa_μ (0.54 parts per million [100]) $\sim 2-3\ \sigma$ opens a window for new physics.

A charged particle with spin $\boldsymbol{s}$ has a magnetic moment

$$\boldsymbol{\mu_\mu} = g(\frac{e}{2m}\boldsymbol{\mu}), \quad a_\mu \equiv \frac{(g-2)}{2}, \quad \mu_\mu = (1+a_\mu)\frac{e\hbar}{2m} \tag{6.9}$$

where the Dirac equation predicts the gyromagnetic ratio $g_\mu = 2$ for point-like, spin 1/2 particles. The quantity $g-2$ probes the difference between the mass and the charge distribution of a particle and $g-2=0$ when they are the same at all times, a_μ is the anomaly.

The SM contributions come from three types of radiative processes: QED loops containing leptons (e, μ, τ) and photons [154], hadronic loops containing hadrons in vacuum polarization loops [155], higher order [156] and weak loops involving the W and Z weak gauge bosons

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{had} + a_\mu^{EW} \tag{6.10}$$

In the 2HDM(II) for the CP conserving case, the neutral scalars h and H, pseudoscalar A as well as the charged boson $H^\pm$ can contribute to a_μ, for one loop level see Refs. [63,157, 158] and for two-loop level see Refs. [159–161]. The light Higgs boson, h or A dominates the full 2HDM(II) contributions, i.e., $a_\mu^{2HDM} \approx a_\mu^h$ or a_μ^A [162]. The scalar contribution a_μ^h is positive whereas the pseudoscalar and the charged Higgs boson contributions are negative [163, 164].

The deviation of the SM theory from the experimental value of the anomalous magnetic moment of the muon opens a window for the 2HDM contributions. This deviation can be written as

$$\Delta a_\mu = a_\mu^{expt} - a_\mu^{SM} \tag{6.11}$$

According to Refs. [165, 166], for the Higgs mass heavier than 3 GeV the dominant Higgs contributions to a_μ come from the two-loop Barr-Zee effect [169] with a photon and Higgs field connected to a heavy fermion (f) running in the inner loop. The one-loop and the two-loop contributions have different signs for both scalar and pseudoscalar contributions, therefore the one-loop is partially cancelled by the larger two-loop contributions [167].

The contributions from all the neutral Higgs boson at one loop level for the CP conserving case are given by (lepton contributions) [168]

$$\Delta a_\mu = \frac{m_\mu m_\tau}{16\pi^2}\sum_i b_i^2\Big[N(m_{H_i}) + \frac{m_\mu}{3m_\tau}M(m_{H_i})\Big] + a_i^2\Big[N(m_{H_i}) - \frac{m_\mu}{3m_\tau}M(m_{H_i})\Big] \tag{6.12}$$

with

$$M(m_{H_i}) = \Big[\frac{2+3(\frac{m_\tau}{m_{H_i}})^2 + 6(\frac{m_\tau}{m_{H_i}})^2\ln(\frac{m_\tau}{m_{H_i}})^2 - 6(\frac{m_\tau}{m_{H_i}})^4 + (\frac{m_\tau}{m_{H_i}})^6}{m_{H_i}^2\big(1-(\frac{m_\tau}{m_{H_i}})^2\big)^4}\Big] \tag{6.13}$$

$$N(m_{H_i}) = \Big[\frac{3+(\frac{m_\tau}{m_{H_i}})^2\big((\frac{m_\tau}{m_{H_i}})^2-4\big)+2\ln(\frac{m_\tau}{m_{H_i}})^2}{m_{H_i}^2\big(1-(\frac{m_\tau}{m_{H_i}})^2\big)^3}\Big]\frac{m_\tau}{m_{H_i}}, \quad i = m_h, m_H, m_A \tag{6.14}$$

where a_i and b_i are the coefficients of the Feynman rule for the scalar and pseudoscalar Higgs bosons, respectively.

The two loop level contributions with a heavy fermion f to a_μ from all neutral Higgs bosons in the CP conserving case take the form [160]

$$\Delta a_\mu = \frac{N_c\alpha_{\text{e.m.}}}{4\pi^3 v^2}\, m_\mu^2 Q_f^2\Big[A_l A_f g(\frac{m_f^2}{m_A^2}) - \lambda_l\lambda_f f(\frac{m_f^2}{m^2})\Big] \tag{6.15}$$

where A_l and A_f are the coupling constants for pseudoscalar to leptons and fermions respectively, λ_l and λ_f are the coupling constants for scalar to leptons and fermions respectively, $N_c = 3$ the number of colours associated with the fermion loop, $\alpha_{\text{e.m.}}$ the electromagnetic finestructure constant, Q_f is the fermion charge, $m_W^2\sin^2\theta_W = (\pi\alpha)/(\sqrt{2}G_F)$ and $v^2 = 1/\sqrt{2}G_F$. The functions f and g are given in [169]:

$$f(z) \equiv \frac{1}{2}z\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-z}\ln\frac{x(1-x)}{z} \tag{6.16}$$

$$g(z) \equiv \frac{1}{2}z\int_0^1 dx\frac{1}{x(1-x)-z}\ln\frac{x(1-x)}{z} \tag{6.17}$$

where

$$\begin{aligned} f(1) &\sim \frac{1}{2}, \quad g(1)\sim 1, \\ f(z) &\sim \frac{1}{3}\ln z + \frac{13}{18}, \quad g(z)\sim\frac{1}{2}\ln z + 1 \quad \text{for large} \quad z \\ f(z) &\sim g(z) \sim \frac{z}{2}(\ln z)^2 \quad \text{for small} \quad z \end{aligned} \tag{6.18}$$

The contribution of the 2HDM in the case of CP violation can be found in Ref. [17] for the top quark contribution:

$$\Delta a_\mu = \frac{N_c\alpha_{\text{e.m.}}}{4\pi^3 v^2}\, m_\mu^2 Q_t^2 \sum_j\left[R_{j3}^2\, g\left(\frac{m_t^2}{M_j^2}\right) - \frac{1}{\cos\beta\sin\beta}\, R_{j1}R_{j2}\, f\left(\frac{m_t^2}{M_j^2}\right)\right], \tag{6.19}$$

with $Q_t = 2/3$ and m_t the top quark charge and mass, R_{ji} are the rotation matrix elements and m_μ the muon mass. It is worth noting that the $\tan\beta$ factor associated with the pseudoscalar Yukawa coupling of the muon is cancelled by an opposite factor associated with the top quark. While the first term gives a positive contribution, the second one may have either sign.

The contribution of the b quark can be obtained from (6.19) by trivial substitutions for Q_t and m_t accompanied by

$$R_{j3}^2 \to \tan^2\beta R_{j3}^2, \quad \text{and} \quad \frac{1}{\cos\beta\sin\beta}R_{j1}R_{j2} \to \frac{1}{\cos^2\beta}R_{j1}^2 \tag{6.20}$$

in the square bracket. At low values of $\tan\beta$, the contributions of the 2HDM are negligible, but the b- and τ-loop contributions can become relevant at large $\tan\beta$ and low values of $m_{H^\pm}$ [17], but the unitarity constraint excludes the large values of $\tan\beta$. So we conclude that the effect of the anomalous magnetic moment is not important. For more details see Ref. [17].

The a_μ constraint does not have any significant impact within the range of $\tan\beta$ considered [19].

The contributions of the 2HDM to the ρ parameter will be studied in the next section.

6.3 The Electroweak Correction to the Parameter ρ

Electroweak precision measurements and calculations provide stringent and decisive tests of the quantum fluctuations predicted from the theory [172]. The electroweak parameter ρ is a measure of the relative strength of neutral and charged-current interactions in four-fermions process at zero momentum transfer. In the SM at tree level the ρ parameter relates the W gauge boson and Z gauge boson masses as

$$\rho = \frac{m_W^2}{c_W^2 m_Z^2} = 1 \tag{6.21}$$

The SM correction to ρ at the one-loop level is induced by the top quark [173]

$$\rho_{top} = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \tag{6.22}$$

The dominant contributions in the large top mass limit come from corrections to the W and Z boson propagator involving the t- and b-quark loops.

The higher order loop corrections modify the ρ parameter into

$$\rho = \frac{1}{1-\Delta\rho} \tag{6.23}$$

Here $\Delta\rho$ parametrizes all higher loop corrections which are sensitive to the existence of heavy particles in the SM, in particular the top quark and the Higgs boson. The leading one-loop calculations (comes only from self energy corrections to the gauge boson propagators and not from the vertex or box diagrams) depend logarithmically on m_H [174]. The leading two-loop corrections, which are proportional to m_H^2, have been calculated in Refs. [177, 178]. The leading three-loop bosonic corrections to the ρ parameter proportional to m_H^4 in the large Higgs mass limit are calculated in Ref. [179] and four loop corrections have been calculated in [172]. The corrections coming from the transversal W and Z self-energies can be written as [180, 181]:

$$\Delta\rho \equiv \frac{1}{M_W^2}\left[A_{WW}(q^2=0) - \cos^2\theta_W A_{ZZ}(q^2=0)\right] \tag{6.24}$$

and measures how much the W and Z self-energies can deviate from the Standard-Model value, being zero at the tree level.

In the 2HDM additional contributions arise [67], which are determined by the couplings to the W and the Z of the Higgs particles, and by the mass splittings within the Higgs sector, as well as the mass splittings with respect to the W and Z bosons.

The contributions of the 2HDM to the ρ parameter in the CP conserving case can be written as [13, 164, 182]

$$\begin{aligned}\Delta\rho^{2HDM} &= \frac{\alpha}{16\pi s_W^2 c_W^2}\Big(\cos^2(\beta-\alpha)\Big[f(m_A,m_{H^\pm})+f(m_{H^\pm},m_h)-f(m_A,m_h)\Big]\\ &\quad+\sin^2(\beta-\alpha)\Big[f(m_A,m_{H^\pm})+f(m_{H^\pm},m_H)-f(m_A,m_H)\Big]\Big)\\ &\quad+\cos^2(\beta-\alpha)\Delta\rho_{SM}(m_H)+\sin^2(\beta-\alpha)\Delta\rho_{SM}(m_h) \end{aligned} \tag{6.25}$$

with

$$f(x,y)=\frac{x^2+y^2}{2}-\frac{x^2y^2}{x^2-y^2}\ln\frac{x^2}{y^2} \tag{6.26}$$

This function is symmetric in $x\leftrightarrow y$, and vanishes for $x=y$, $f(x,y)$ is large for large difference between x and y. Furthermore,

$$\Delta\rho_{SM}(m)=-\frac{3\alpha}{16\pi s_W^2 c_W^2}\Big[f(m,m_W)-f(m,m_Z)\Big]-\frac{\alpha}{8\pi c_W^2} \tag{6.27}$$

where m is the SM Higgs mass.

The simplified forms provided in [13] for the 2HDM can easily be re-expressed in terms of the mass eigenvalues and the elements R_{jk} of the rotation matrix for the CP-violating case. The Higgs–Higgs contributions to the parameter ρ in the CP violation are modified to [17, 19]

$$\begin{aligned}&A_{WW}^{HH}(0)-\cos^2\theta_W\, A_{ZZ}^{HH}(0)\\ &=\frac{g^2}{64\pi^2}\sum_j\Big[\{[\sin\beta R_{j1}-\cos\beta R_{j2}]^2+R_{j3}^2\}F_{\Delta\rho}(M_{H^\pm}^2,M_j^2)\\ &\quad-\sum_{k>j}[(\sin\beta R_{j1}-\cos\beta R_{j2})R_{k3}-(\sin\beta R_{k1}-\cos\beta R_{k2})R_{j3}]^2\,F_{\Delta\rho}(M_j^2,M_k^2)\Big],\end{aligned} \tag{6.28}$$

For the Higgs–ghost contribution, we have to subtract the contribution from a Standard-Model Higgs of mass M_0, since this is already taken into account in the fits, and find:

$$\begin{aligned}&A_{WW}^{HG}(0)-\cos^2\theta_W\, A_{ZZ}^{HG}(0)\\ &=\frac{g^2}{64\pi^2}\Big[\sum_j[\cos\beta\, R_{j1}+\sin\beta\, R_{j2}]^2\Big(3F_{\Delta\rho}(M_Z^2,M_j^2)-3F_{\Delta\rho}(M_W^2,M_j^2)\Big)\\ &\quad+3F_{\Delta\rho}(M_W^2,M_0^2)-3F_{\Delta\rho}(M_Z^2,M_0^2)\Big].\end{aligned} \tag{6.29}$$

From the electroweak fits, we take $M_0=129$ GeV, but note that this value is not very precise [183].

In order to keep these additional contributions (6.28) and (6.29) small, the charged Higgs boson should not be coupled too strongly to the W if its mass is far from those of its neutral partners.

In the following figures, we use the approach of Paper 2 [19] for calculating the $\Delta\rho$ constrant. We show the implication of the $\Delta\rho$ constraint for the case when M_3 is calculated from the other input, i.e., M_3 is calculated from the relation (4.45).

Here, the calculations for the $\Delta\rho$ constraint are different from the calculations made in our paper of Ref. [19] as follows:

- Here we use $M_1 = 150$ GeV (greater than the lower limit of the SM Higgs $m_H \simeq$ 114 GeV), in Ref. [19] we use $M_1 = 100$ GeV.
- We use in this study some new values of μ^2, such as (400 GeV)^2, $-$(300 GeV)^2, and $-$(400 GeV)^2.

We will next show some figures for the $\Delta\rho$ constraint, and select the following values for the parameters M_1, M_2, M_3 and μ^2 as:

- $M_1 = 150$ GeV.
- M_2 =(300, 500) GeV.
- $\mu^2 = (400\text{ GeV})^2, (300\text{ GeV})^2, 0, -(300\text{ GeV})^2, -(400\text{ GeV})^2$.

The unitarity constraint and $\Delta\rho$ constraints exclude larger values of $\tan\beta$ for different values of our parameters such as M_2 and μ^2 than in the case of Paper 2 ($M_1 = 100$ GeV) [19].

The upper limit on $\tan\beta$ and charged Higgs mass $m_{H^\pm}$ become smaller as M_1 increases, compare fig. 7 in Ref. [19].

Figs. 6.11 and 6.12 are devoted to $\mu = 400$ GeV, with two values of M_2, 300 and 500 GeV. In fig. 6.11 the higher values of the charged Higgs mass $M_{H^\pm}$ are excluded at low $\tan\beta$.

In fig. 6.12 μ is comparable with M_2 then, there can be a considerable splitting between M_2 and M_3, $M_{H^\pm}$ and M_3 which can be similar and a cancellation between the (M_1, $M_{H^\pm}$) and the (M_1, M_3) terms of Eq.(4.12) in Ref. [19] is possible. As a result, for large values of μ, large values of $M_{H^\pm}$ can be allowed. But low values of $M_{H^\pm}$ are forbidden at large $\tan\beta$, see condition (4.17) in Ref. [19].

The μ that is used in figs. 6.13 and 6.14 is 300 GeV. The upper limit on the charged Higgs mass $M_{H^\pm}$ decreases as $\tan\beta$ increases, see fig 6.13.

In figs 6.15 and 6.16 we consider $\mu = 0$ GeV. The contribution of $\Delta\rho$ in fig. 6.15 is similar to fig. 7 in Ref. [19] but here the model is more constrained.

Figs. 6.17 and 6.18 are devoted to $\mu^2 = -(300\text{ GeV})^2$. The allowed region of the parameter space in the 2HDM is here dramatically reduced and the upper limit of $\tan\beta$ (the cutoff will be at $\tan\beta \sim 2$) and charged Higgs mass ($m_{H^\pm} \sim 500$ GeV) are also reduced. For negative values of μ^2 the unitarity constraint excludes large $\tan\beta$, large values of the charged Higgs mass and also low values of $\tan\beta$.

The μ^2 that is used in figs. 6.19 and 6.20 is $-$(400 GeV)^2, it leads to more exclusion of the parameter space than in figs. 6.17 and 6.18. In fig. 6.20 the whole parameter space is excluded.

We conclude that for large values of M_1 more exclusion of the parameter space is done. The negative values of μ^2 exclude most of the parameter space due to unitarity and also

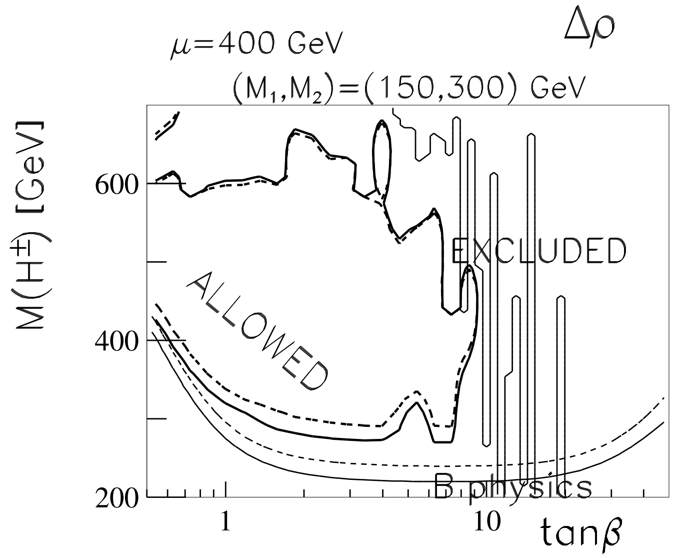

Figure 6.11: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

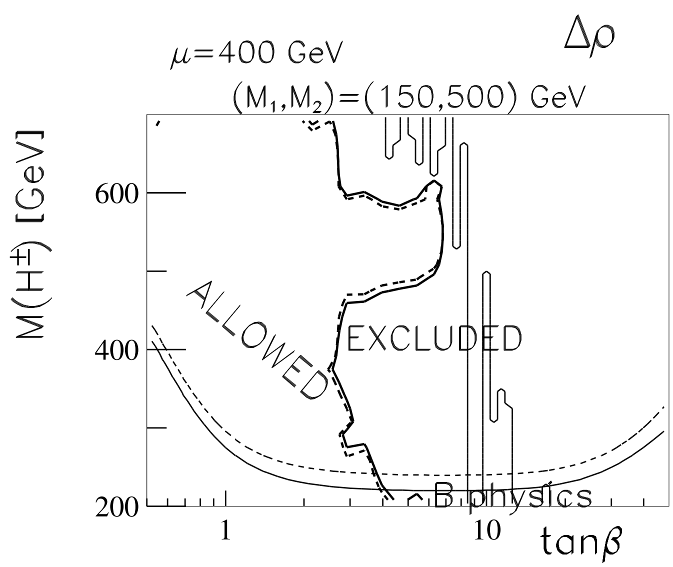

Figure 6.12: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

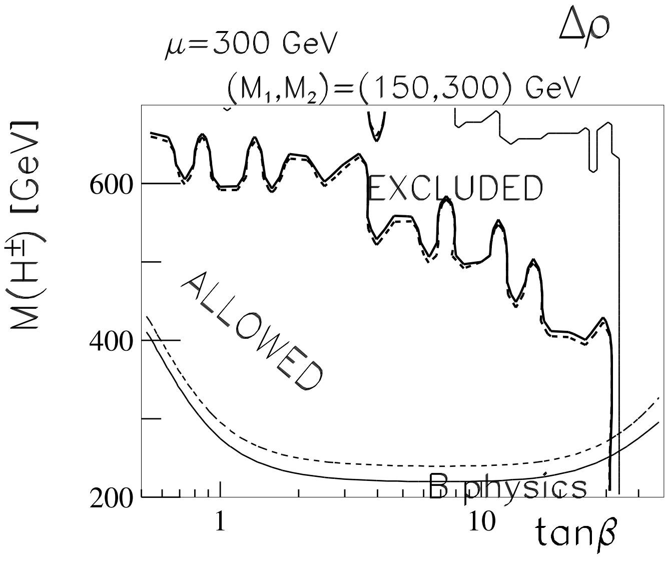

Figure 6.13: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

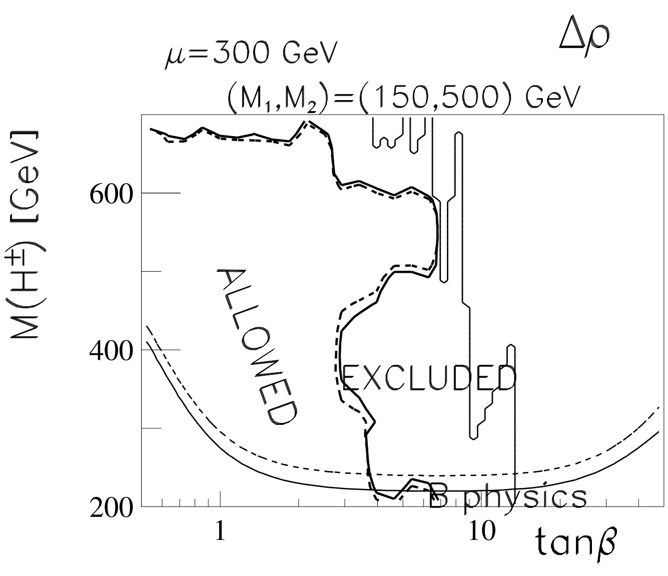

Figure 6.14: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

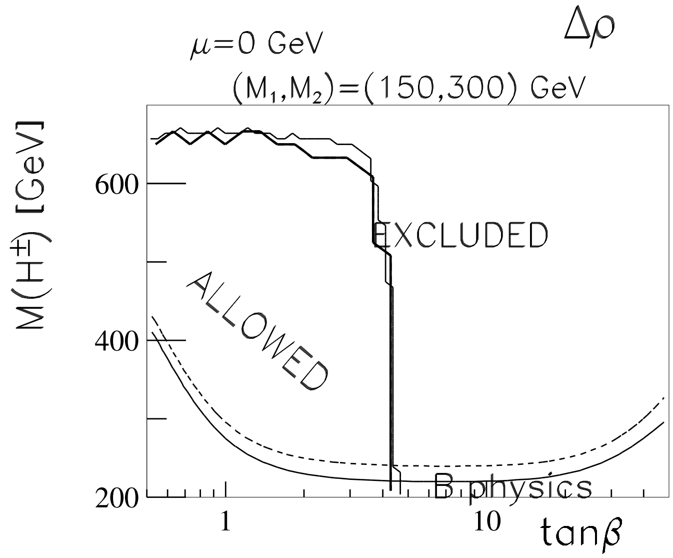

Figure 6.15: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

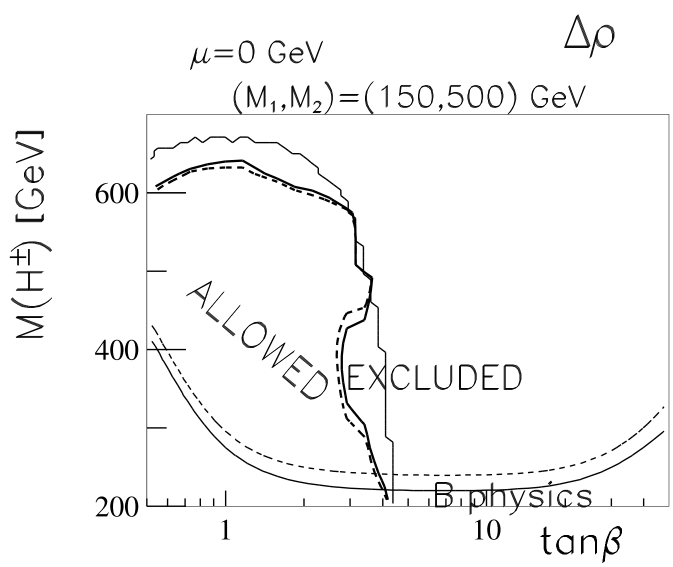

Figure 6.16: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

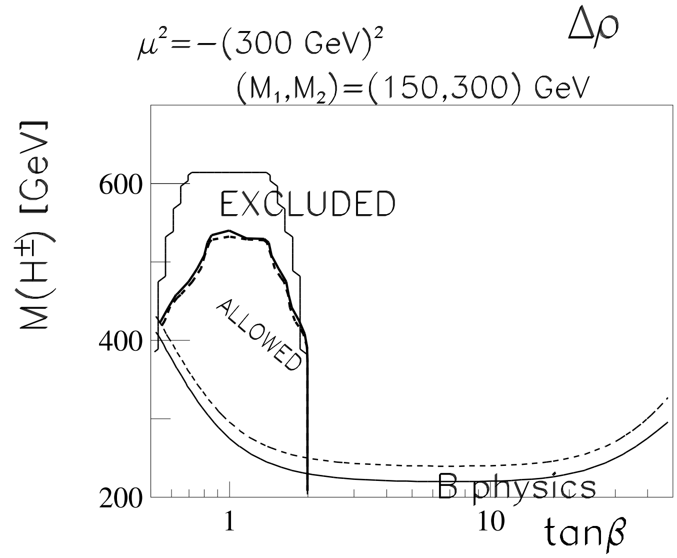

Figure 6.17: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

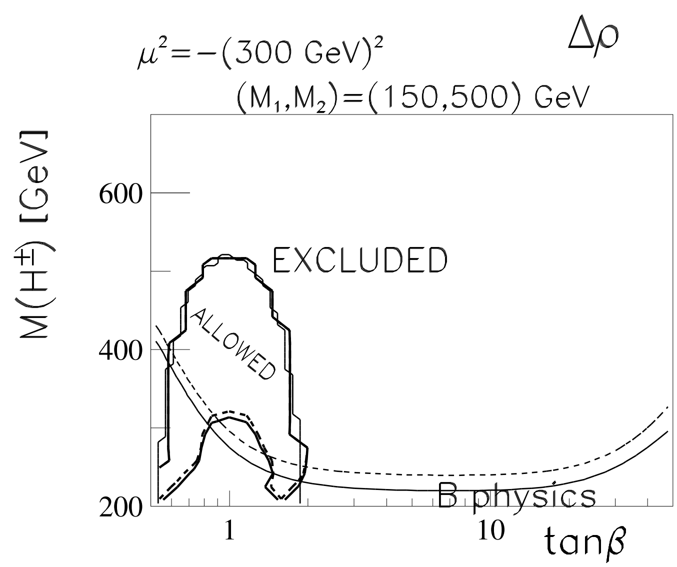

Figure 6.18: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

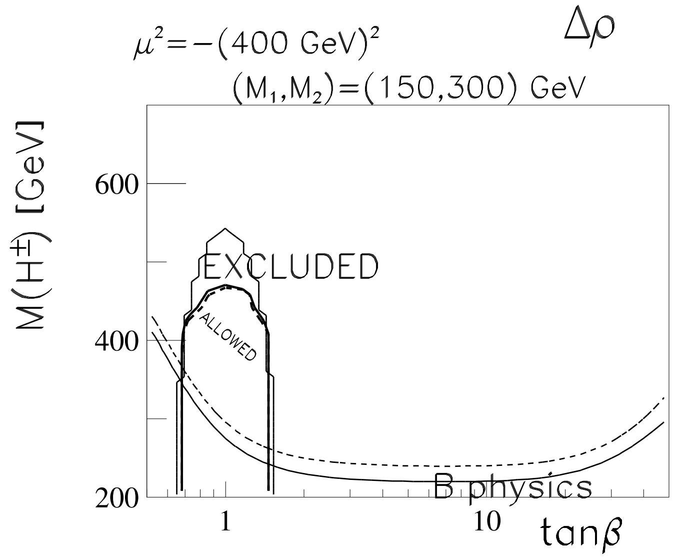

Figure 6.19: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

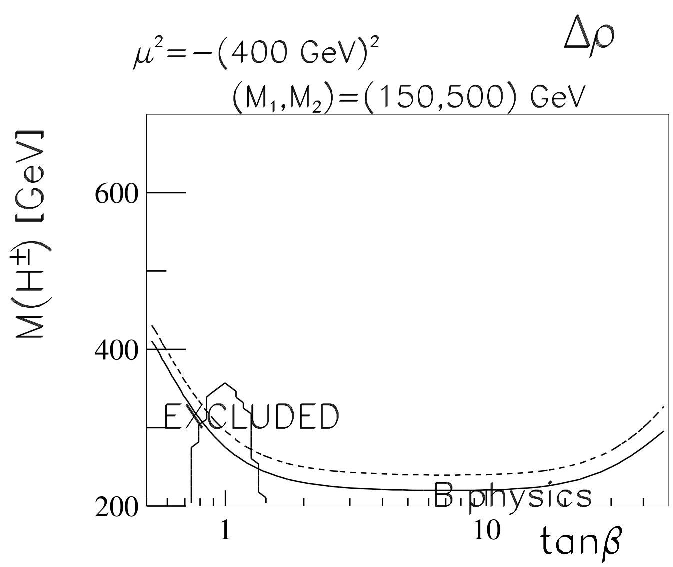

Figure 6.20: Excluded regions due to constraints from positivity, unitarity, $\Delta\rho$, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

exclude the low values of $\tan\beta$. At large enough negative values of μ^2 and at large M_2 the model will be inconsistent.

In the next section we will study the LEP2 constraint.

6.4 The LEP2 constraint

The analysis combining LEP1 and LEP2 data collected with OPAL are sensitive to the h, $A \to q\bar{q}, \tau^+\tau^-$ and $h \to AA$ decay modes of the Higgs boson [184]. At LEP2, the Higgs bosons can be produced through the Higgs-strahlung process $e^+e^- \to HZ$, also it is sensitive to the contributions to the signal from the W^+W^- and ZZ fusion processes $H\nu\bar{\nu}$ and He^+e^-. There are three main processes for Higgs production at LEP: the Higgs-strahlung (Bjorken) processes $e^+e^- \to hZ$ and HZ, the pair production processes $e^+e^- \to hA$ and HA processes ($e^+e^- \to hH$ is forbidden by the angular momentum and CP conservation), and the Yukawa radiation off heavy fermions $e^+e^- \to f\bar{f}h$ and $f\bar{f}A$. The first two, gauge-mediated processes are bounded by the SM hZ cross-section, mixing of the Higgs doublets induces partial or total suppression with respect to the SM. The last, fermion-mediated process can be significantly enhanced compared to the SM $f\bar{f}h$ cross-section, which is too low to be observed at LEP. The coupling of the pseudoscalar A to fermions can be written as [185]

$$Ab\bar{b}: \quad -i\gamma_5 \tan\beta \tag{6.30}$$

$$At\bar{t}: \quad -i\gamma_5 \cot\beta \tag{6.31}$$

In the analysis of LEP data by DELPHI, a channel-specific dilution factor C^2 is defined by [186]

$$\sigma_{Zh\to b\bar{b}Z} = \sigma_{Zh}^{\rm SM} \times C^2_{Z(h\to bb)}, \tag{6.32}$$

$$\sigma_{Zh\to\tau^+\tau^-Z} = \sigma_{Zh}^{\rm SM} \times C^2_{Z(h\to\tau\tau)}, \tag{6.33}$$

The reference cross-section for the process $e^+e^- \to hA$ is obtained by computing this process in the absence of any mixing in the Higgs sector, and depends only on the electroweak constants and h and A Higgs masses. So that the pair production process for the 2HDM can be written in terms of the reference cross-section as [186]

$$\sigma_{hA\to 4f} = \sigma_{hA}^{\rm ref} \times C^2_{hA\to 4f}, \tag{6.34}$$

$$\sigma_{(AA)A\to 6b)} = \sigma_{hA}^{\rm ref} \times C^2_{hA\to 6b}, \tag{6.35}$$

The reference cross-section for the Yukawa process is obtained in a similar way. The SM $e^+e^- \to f\bar{f}h$ ($f = b, \tau$) cross-section is used for h production. Computing this cross-section with a suitable $f\bar{f}A$ vertex gives the reference for A, therefore

$$\sigma_{b\bar{b}h\to 4b} = \sigma_{bbh}^{\rm SM} \times C^2_{bb(h\to bb)}, \tag{6.36}$$

$$\sigma_{b\bar{b}h\to b\bar{b}\tau^+\tau^-} = \sigma_{bbh}^{\rm SM} \times C^2_{bb(h\to\tau\tau)}, \tag{6.37}$$

In the CP-violating case, the H_1ZZ coupling is suppressed by the square of the Higgs-vector-vector coupling, which relative to the Standard-Model coupling is

$$H_jZZ: \qquad [\cos\beta R_{j1} + \sin\beta R_{j2}], \quad \text{for } j = 1. \tag{6.38}$$

For the decaying of H_1 to $b\bar{b}$ or $\tau\bar{\tau}$, the dilution is caused by the two effects discussed above: There is a reduction of the coupling to the Z boson [see (6.38)] and modified (typically reduced) coupling to the $b\bar{b}$ (or $\tau\bar{\tau}$). Thus, we take [17]

$$C^2_{Z(H_1\to b\bar{b})} = [\cos\beta R_{11} + \sin\beta R_{12}]^2 \, \frac{1}{\cos^2\beta} \, [R_{11}^2 + \sin^2\beta R_{13}^2], \tag{6.39}$$

and consider as excluded parameter sets those where this quantity exceeds the LEP bounds, roughly approximated as [186]

$$C^2_{Z(H_1\to b\bar{b})} = 0.2 \quad \text{at 100 GeV}, \quad \text{and 0.1 at 80 GeV}. \tag{6.40}$$

In the following figures, we use the approach of Paper 1 [17] for calculating the LEP2 constraint. We show the implication of the LEP2 constraint for the case when M_3 is calculated from the other input, i.e., M_3 is calculated from the relation (4.45).

The calculations for the LEP2 constraint are different from the calculations made in our paper of Ref. [19] as follows:

- We take M_3 fixed at 500 GeV.
- Here we use $M_1 = 80$ GeV, in Ref. [19] we use $M_1 = 100$ GeV.
- We use in this study some new values of μ^2, such as (400 GeV)^2, (300 GeV)^2 and $-$(400 GeV)^2.

We will next show some figures for the LEP2 constraint, and select the following values for the parameters M_1, M_2, M_3 and μ^2 as:

- $M_1 = 80$ GeV.
- $M_2 =$(300, 500) GeV.
- $M_3 = 500$ GeV.
- $\mu^2 = (400\text{ GeV})^2, (300\text{ GeV})^2, 0, -(300\text{ GeV})^2, -(400\text{ GeV})^2$.

The unitarity constraint excludes the large values of $\tan\beta$ for different values of our parameters such as M_2 and μ^2, see figs 6.21–6.25.

For large values of M_2, (i.e., $M_2 = 500$) and for large positive values of μ^2, the LEP2 constraint excludes the whole parameter space that is not already excluded by the unitarity for large values of M_2, (i.e., $M_2 = 500$) and for large positive values of μ^2, see fig. 6.21. At large negative values of μ^2 there is no allowed region of the parameter space for low values of M_2, as shown in fig. 6.22. Also at large values of M_2, the whole region of the parameter space is excluded (this is not shown).

In figs. 6.23–6.25, at low values of M_2 there is no exclusion when $\mu^2 > M_2^2$, see fig. 6.23 and there is a little exclusion when $M_2^2 \gtrsim \mu^2$, see figs. 6.24 and 6.25.

In summary, the LEP2 constraint excludes more parameter space at low values of M_1 (i.e., $M_1 = 80$ GeV) than the case of large value in Ref. [19] ($M_1 = 100$ GeV). The negative values of μ^2 exclude all the allowed regions of the parameter space due to LEP2 at low or large values of M_2, and also at high values of μ^2 and large values of M_2^2 the model is inconsistent.

The combinations of all constraints will be disussed in the next Chapter.

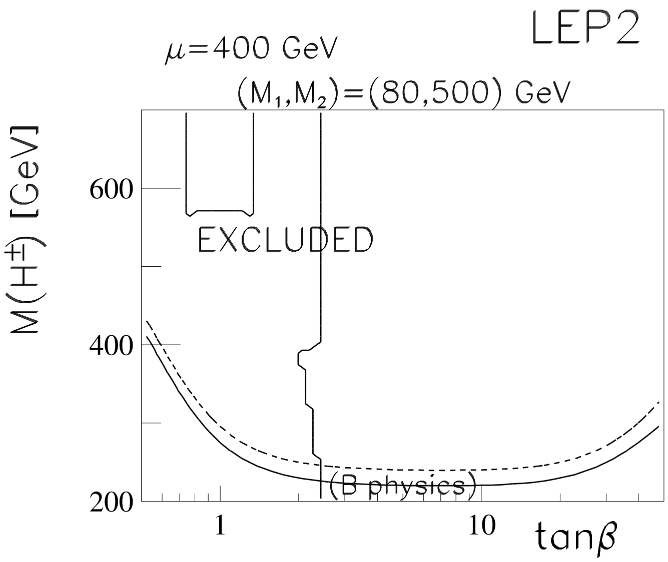

Figure 6.21: Excluded regions due to constraints from positivity, unitarity, LEP2, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

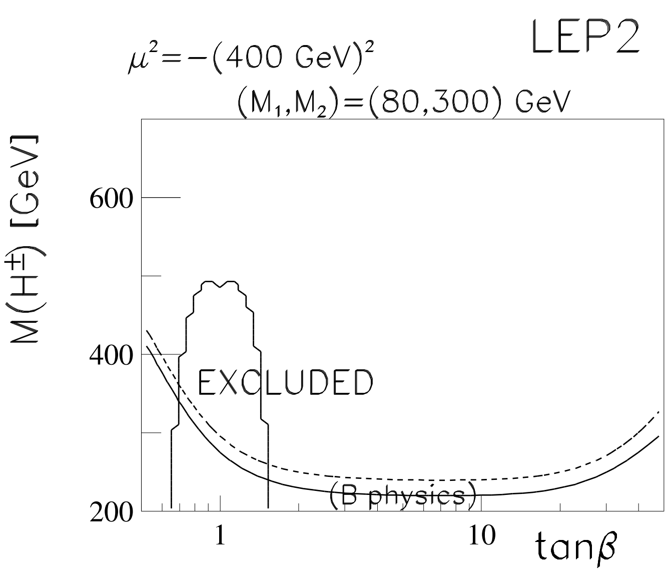

Figure 6.22: Excluded regions due to constraints from positivity, unitarity, LEP2, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

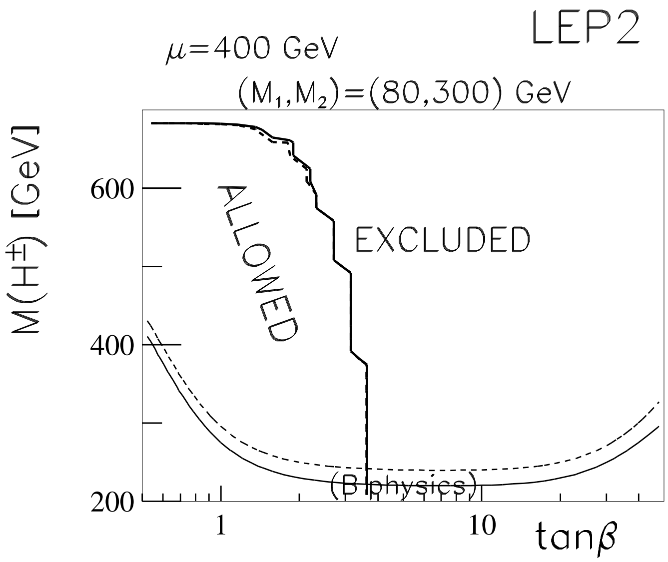

Figure 6.23: Excluded regions due to constraints from positivity, unitarity, LEP2, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

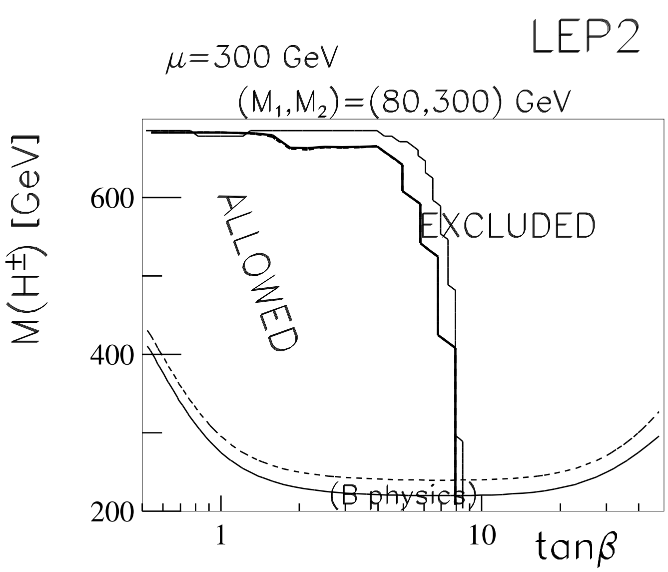

Figure 6.24: Excluded regions due to constraints from positivity, unitarity, LEP2, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

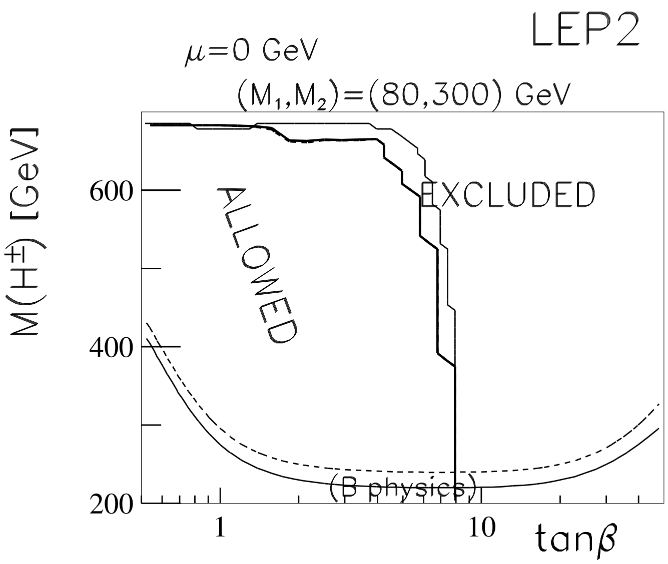

Figure 6.25: Excluded regions due to constraints from positivity, unitarity, LEP2, and $B \to X_s\gamma$. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

Chapter 7

Combining all constraints

Let us now combine all constraints and look for the allowed points in the $\tan\beta$–$M_{H^\pm}$ plane. This work is different from the calculations made in our paper of Ref. [19] as follows:

- We take M_3 fixed at 500 GeV, whereas in Ref. [19] M_3 is calculated from the other input, i.e., M_3 is calculated from the relation (4.45). Therefore, we can not directly compare results here with those in [19].

- Here we use $M_1 = 150$ GeV (greater than the lower limit of the SM Higgs $m_H \simeq$ 114 GeV), in Ref. [19] we use $M_1 = 100$ GeV.

- We use in this study some new values of μ^2, such as $(400\text{ GeV})^2$, $-(300\text{ GeV})^2$, and $-(400\text{ GeV})^2$.

We will next show some figures for the combining of all constraints, and select the following values for the parameters M_1, M_2, M_3 and μ^2 as:

- $M_1 = 150$ GeV.

- $M_2 =$(300, 500) GeV.

- $M_3 = 500$ GeV.

- $\mu^2 = (400\text{ GeV})^2, (300\text{ GeV})^2, 0, -(300\text{ GeV})^2, -(400\text{ GeV})^2$.

The unitarity constraint excludes large values of $\tan\beta$ for different values of our parameters such as M_2 and μ^2, see figs. 7.1–7.10. There are exclusions coming from the combinations all constraints, i.e., the $\Delta\rho$ constraint excludes the large values of the charged Higgs mass at low values of $\tan\beta$ and excludes low values of the charged Higgs mass at large values of $\tan\beta$. The R_b constraint excludes low values of $\tan\beta$ and large values of the charged Higg mass. The a_μ constraint does not have any impact to the parameter space at low values of $\tan\beta$. The LEP2 constraint does not have any role of this exclusion here because of the choice of the mass M_1 greater than the upper experimental limit.

Figs. 7.1 and 7.2 are devoted to $\mu = 400$ GeV, with two values of M_2, 300 and 500 GeV. We notice that the combinations of all constraints at this large value of μ^2 excludes a large part of the parameter space which is not excluded by the the unitarity constraint.

The μ that is used in figs. 7.3 and 7.4 is 300 GeV. The combinations of the constraints have greater impact on the parameter space of the 2HDM than the unitarity constraint, as shown in these figures.

In figs 7.5 and 7.6 we consider $\mu = 0$ GeV. The $\Delta\rho$ constraint excludes the large values of the charged Higgs mass. It is clear to see that for low values of M_2 the exclusion of the large values of the charged Higgs mass is greater than in the case of large values of M_2, see figs. 7.3 and 7.5. In contrast at large values of M_2 the exclusion of large values of $\tan\beta$ is greater than in the case of low values of M_2, see figs. 7.2, 7.4 and 7.6.

Figs 7.7 and 7.8 are devoted to $\mu^2 = -(300 \text{ GeV})^2$. The allowed region of the parameter space in the 2HDM is here dramatically reduced due to unitarity and the combinations of all constraints. The upper limit of $\tan\beta$ (the cutoff will be at $\tan\beta \sim 2$) and charged Higgs mass ($m_{H^\pm} \sim 500$ GeV) becomes smaller for large values of M_2. For negative values of μ^2 the unitarity constraint excludes large values of $\tan\beta$ and also low values of $\tan\beta$.

The μ^2 that is used in figs 7.9 and 7.10 is $-(400\text{GeV})^2$, it leads to more exclusion of the parameter space than at $\mu^2 = -(300 \text{ GeV})^2$. The upper limit on $m_{H^\pm} \sim 300$ GeV becomes smaller for large values of M_2. The whole parameter space is excluded at $M_2 = 500$ GeV, see fig. 7.10.

We conclude that for large values of M_1, M_2 and M_3 there is an additional exclusion of the parameter space with respect to the low values of these parameters. At $M_3 = 600$ GeV, the combinations of the all constraints exclude the low values of the charged Higgs mass. The unitarity constraint excludes the whole parameter space of the 2HDM at large value of M_3, i.e., $M_3 = 700$ GeV. The negative values of μ^2 exclude most of the parameter space due to unitarity and also exclude the low values of $\tan\beta$. At large enough negative μ^2 and at large values of M_2 the whole parameter space is excluded.

In the next chapter we will try to summarize the thesis.

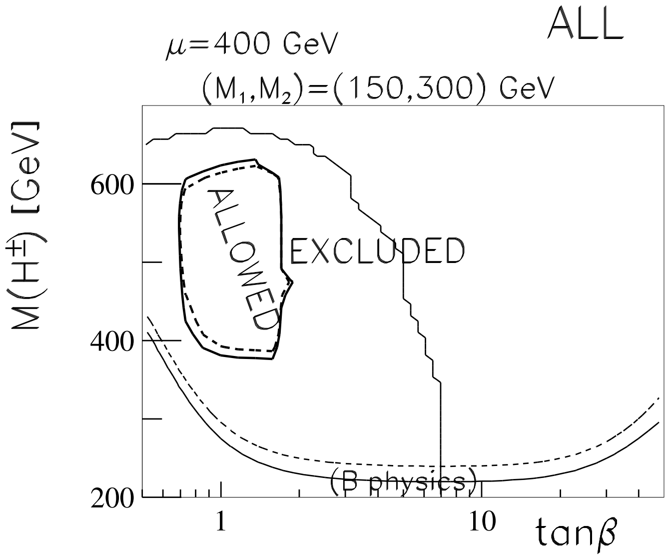

Figure 7.1: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

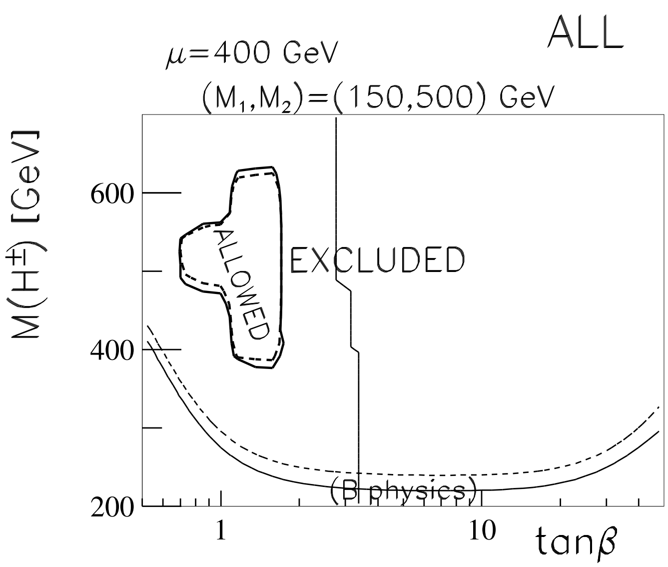

Figure 7.2: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

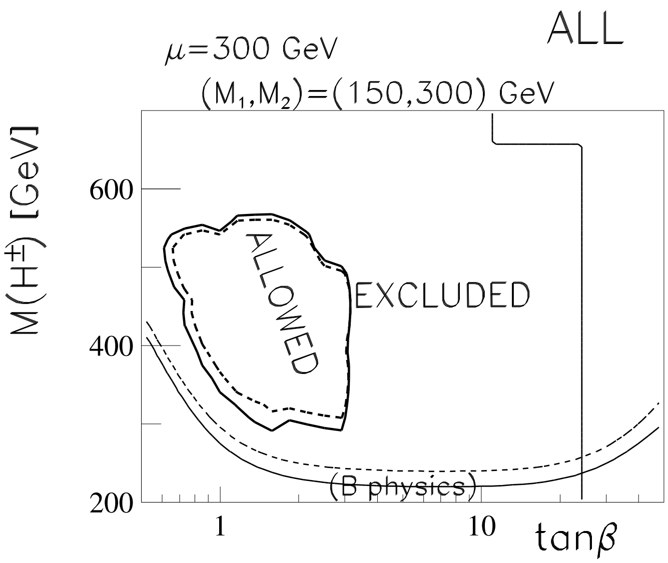

Figure 7.3: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

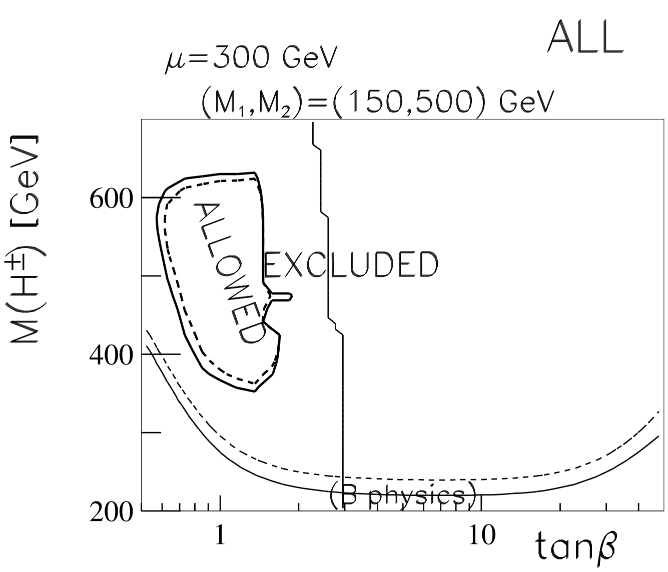

Figure 7.4: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

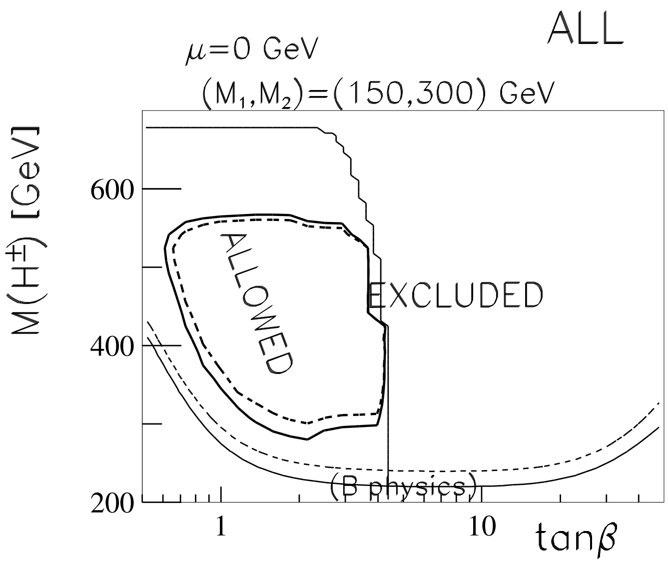

Figure 7.5: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

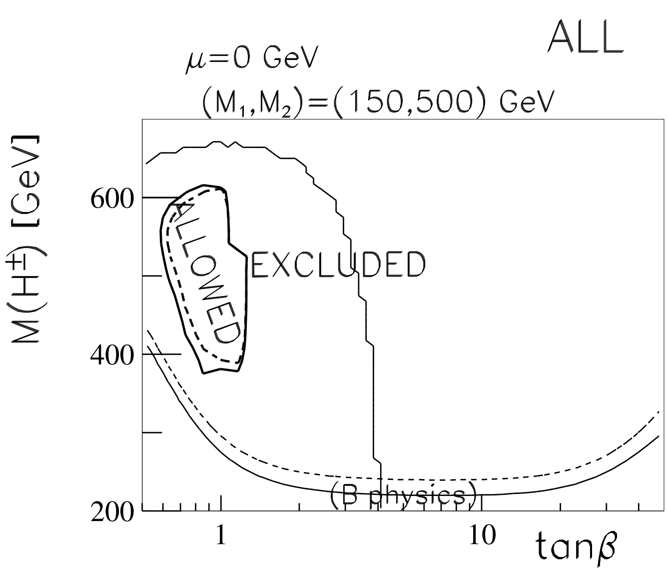

Figure 7.6: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

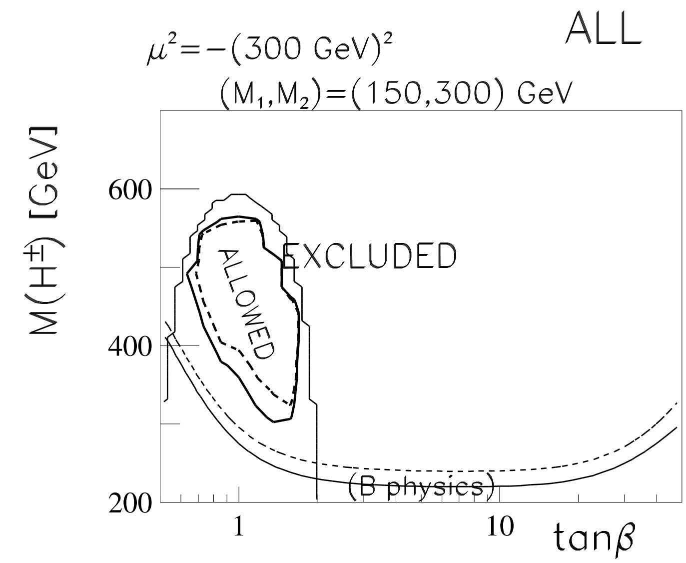

Figure 7.7: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

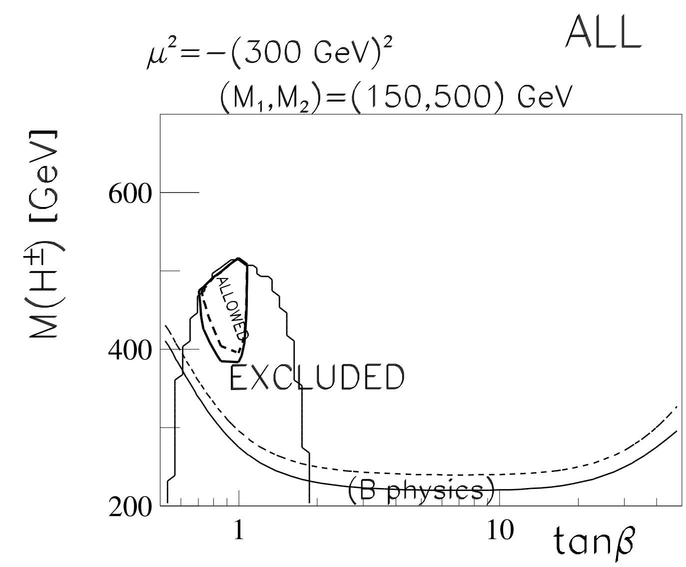

Figure 7.8: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

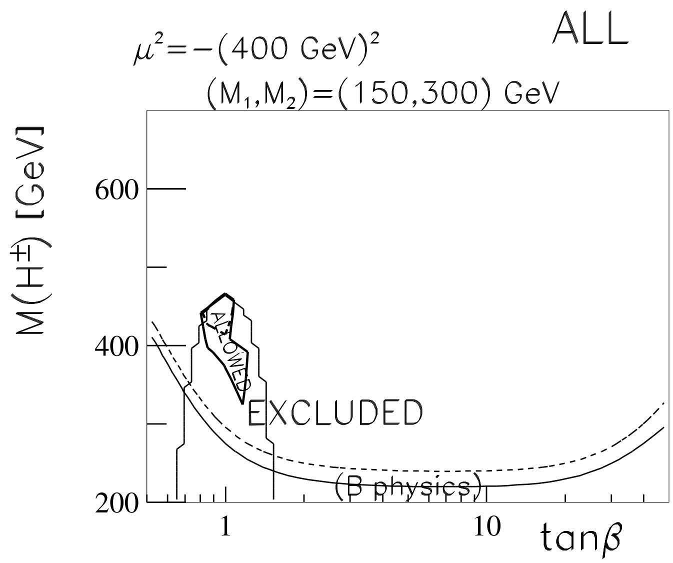

Figure 7.9: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

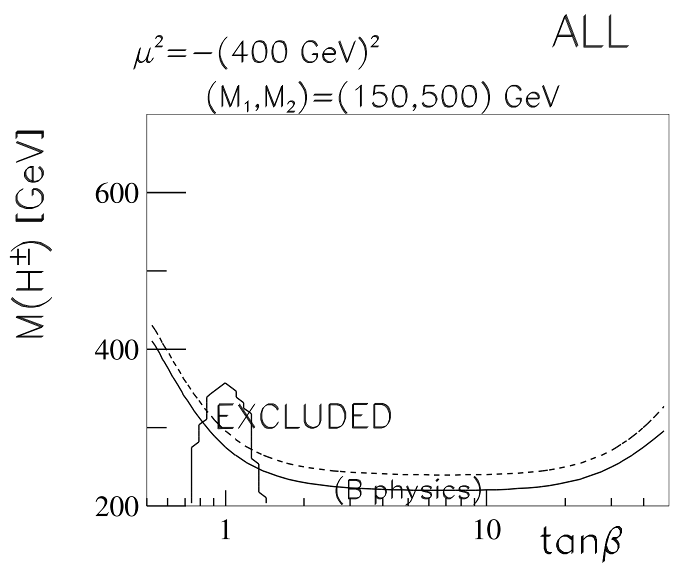

Figure 7.10: Excluded regions due to all constraints. Solid: $\chi^2 = 5$, dashed: $\chi^2 = 3$.

Chapter 8

Summary and conclusion

The thesis presents some studies of the CP-violationg Two-Higgs-Doublet Model. Allowing for the CP violation, an important question is whether this can be measured in the production of top quarks at the LHC. This question is discussed in Paper 1 [17], where we also investigate consistency of the model at low values of $\tan\beta$, where the top Yukawa coupling is large.

In order to maximize the CP violation in the top-quark sector that might be measurable at the LHC, we focus on parameters where (i) the lightest Higgs boson is rather light, in order to maximize the relevant loop integrals, and (ii) where the product of the CP-violating Yukawa couplings, parametrized by $\gamma_{CP}^{(1)}$ [see Eq. (3.63)] are large. The latter constraint requires $\tan\beta$ to be small, and $|\sin\alpha_1 \sin 2\alpha_2|$ to be large.

In the two other papers, we discuss consistency of the model more generally. We have made a survey of parameter space of the Two Higgs Doublet Model. Because of the many independent model parameters, it is difficult to extract a simple picture. We tried in the publications presented in the Appendix A, B, C and D, and in this thesis to scan the whole parameter space of the 2HDM at different values of the following parameters

- Changing the neutral Higgs mass M_1 and display its impact on the 2HDM parameter space. We found that the model becomes less constrained as M_1 increases e.g., at $\mu^2 = (300\text{ GeV})^2$, 0, and $-(300\text{ GeV})^2$.

- Varying the neutral Higgs mass M_2 and see how it constrains the parameter space. We found that the model becomes more constrained as M_2 increases from 300 to 500 GeV.

- The parameter space becomes more constrained as M_3 increases.

- The allowed region of the parameter space becomes large as μ^2 increase to M_2^2, when μ^2 becomes larger than M_2^2 the model becomes more constrained due to the unitarity constraint. For negative values of μ^2, i.e., $\mu^2 = -(300\text{ GeV})^2$, the parameter space of the 2HDM is dramatically reduced due to the unitarity and the combinations of the other constraints.

We investigated the theoretical and experimental constraints individually and the combinations of all for different values of M_1, M_2, M_3 and μ^2 and displayed their impact on the parameter space of the 2HDM. We found the effects of the constraints as follows:

- The positivity and unitarity constraints. For consistency of the model the positivity should be satisfied and also the unitarity constraint. The constraints of positivity and tree-level unitarity exclude most of the multidimensional 2HDM (II) parameter space. The negative values of μ^2 exclude most of the parameter space due to unitarity and also exclude the low values of $\tan\beta$.

- The R_b constraint. For large values of M_1 and M_2, the R_b constraint excludes more parameter space of the 2HDM than in the case of low values of M_1 and M_2. The R_b constraint excludes the low values of $\tan\beta < 1$.

- The a_μ constraint. It does not have any significant impact within the range of $\tan\beta$ considered [19].

- The $\Delta\rho$ constraint. It excludes the large values of the charged Higgs mass at low values of $\tan\beta$ and excludes the low values of the charged Higgs mass at large values of $\tan\beta$. The model becomes constrained as M_1 and M_2 increases. At large enough negative values of μ^2 and at large M_2 the model will be inconsistent.

- The LEP2 non-discovery constraint. It has an impact on the parameter space only for masses $M_1 < 114.4\ GeV$. At large values of μ^2 and large values of M_2^2 the model is inconsistent, see fig. 6.21. Also, the negative values of μ^2 exclude all the parameter space due to LEP2 at low or large values of M_2, this is shown in fig. 6.22

- The $B-\bar{B}$ oscillation constraint. It excludes the low values of $\tan\beta$ and the large values of the charged Higgs mass.

- The $B \to X_s\gamma$ constraint. It excludes the low values of the charged Higgs mass. Improved precision of the $B \to X_s\gamma$ could exclude the remaining part of the parameter space of the 2HDM.

- The $B \to \tau\nu_\tau$ constraint. It excludes the large values of $\tan\beta$, see the right lower corner of all figures in chapters 6 and 7.

- The combinations of all constraints. After combining all constraints, we found that for large values of M_1, M_2 and M_3 there is an additional exclusion of the parameter space with respect to low values of these parameters. At $M_3 = 600$ GeV, the combinations of all constraints exclude the low values of the charged Higgs mass (not shown). The unitarity constraint excludes the whole parameter space of the 2HDM at $M_3 = 700$ GeV. The negative values of μ^2 exclude most of the parameter space due to unitarity and also exclude the low values of $\tan\beta$. At large enough negative μ^2 and at large values of M_2 the whole parameter space is excluded.

We have shown that the B-physics results, together with the precise measurement of the ρ-parameter at LEP and the constraint of tree-level unitarity of Higgs–Higgs scattering, exclude large regions of the 2HDM parameter space. Only moderate values of $\tan\beta \sim 1-5$ and $M_{H^\pm} \sim$ 400–500 GeV remain not excluded. The remaining pockets of allowed parameters will for low values of $\tan\beta$ allow CP violation in the top-quark sector, due to the exchange of Higgs bosons that are mixed with respect to CP.

In summary, we note that even in the face of a variety of experimental constraints, the model is consistent in a number of regions in parameter space. Apart from exceptional points, these allowed regions yield CP violation in $t\bar{t}$ final states produced at the LHC, at a level which can be explored with a data sample of the order of 10^6 semileptonic events.

Improved precision of the $B \to X_s\gamma$ prediction could significantly reduce the remaining part of the parameter space or even exclude the model altogether. The model is inconsistent at $M_3 = 700$ GeV, and for negative enough values of μ^2 with large values of M_2, e.g. $\mu^2 = -(400 \text{ GeV})^2$ and $M_2 = 500$ GeV.

It will be interesting to see how the parameter space would be constrained by future improved constraints from the B-physics sector.

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